Clicker/Poll Question

Suppose we have a two-state system, where the energy of the ground state is 0 and the energy of the excited state is E_0.

What is the partition function for this system? Note $\beta \equiv \frac{1}{kT}$.

A. $Z = 1 + e^{-\beta E_0}$

B. $Z = 1 + \beta E_0$

C. $Z = 1 - \beta E_0$

D. None of the above

E. ???

Clicker/Poll Question

Suppose we have a two-state system, where the energy of the ground state is 0 and the energy of the excited state is E_0.

At what temperature is there a 50% probability that the system is in the ground state?

A. $kT = 0$

B. $kT =$ (some value between 0 and E_0)

C. $kT = E_0$

D. $kT = +\infty$

Clicker/Poll Question

A system has two states of energies $-\epsilon$ and 2ϵ. What is the probability of observing it in the higher energy state at temperature T?

A. 0

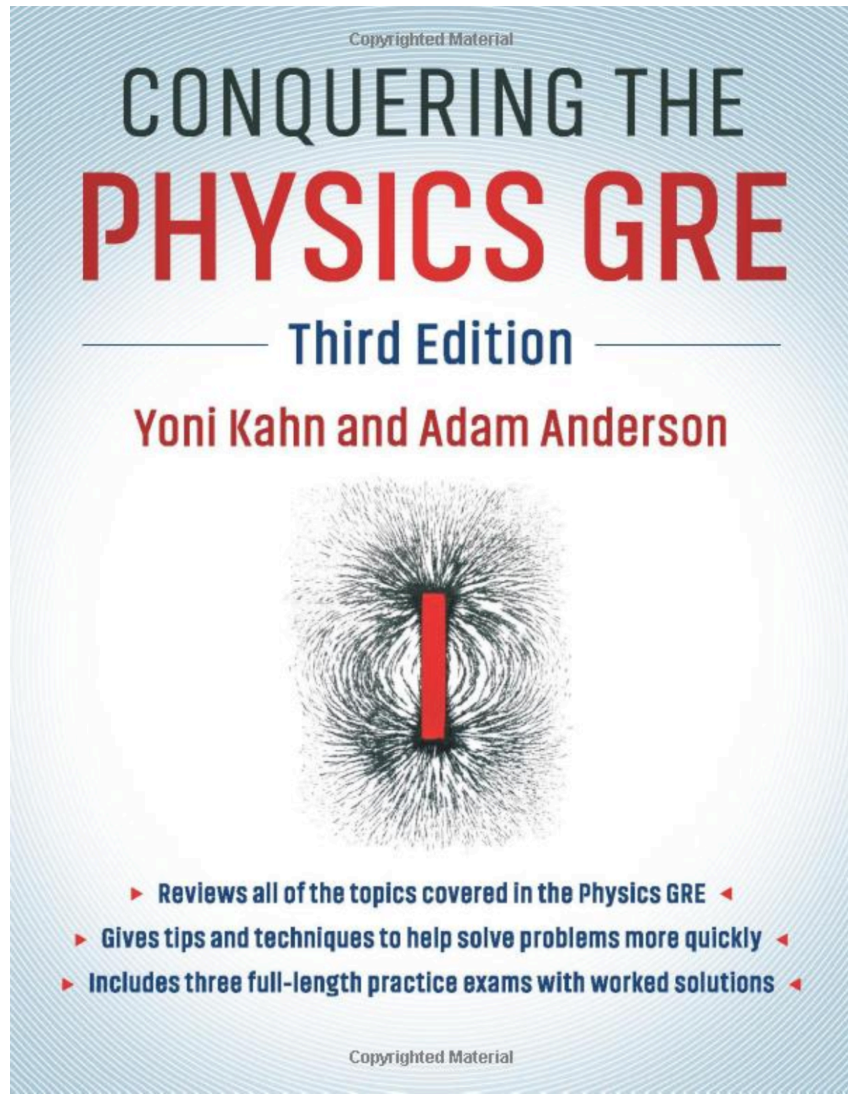
B. $\left[1+\exp\left(\frac{3\epsilon}{kT}\right)\right]^{-1}$

C. $\left[1-\exp\left(\frac{3\epsilon}{kT}\right)\right]^{-1}$

D. $\left[\exp\left(\frac{\epsilon}{kT}\right)+\exp\left(\frac{-2\epsilon}{kT}\right)\right]^{-1}$

E. 1

Problem is from



Clicker/Poll Question

What is the partition function of a one-dimensional quantum harmonic oscillator?

A. $\exp\left(\frac{-\hbar\omega}{kT}\right)$

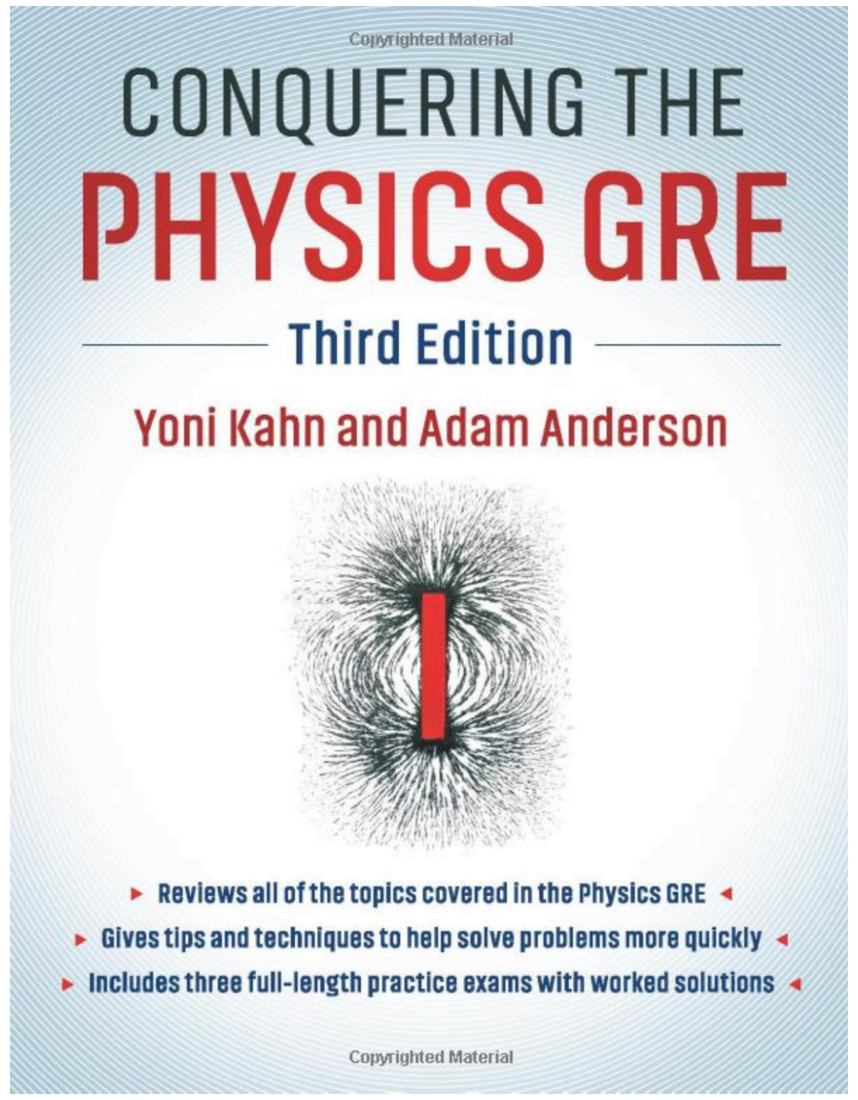
B. $1 - \exp\left(\frac{-\hbar\omega}{kT}\right)$

C. $\left[1 - \exp\left(\frac{-\hbar\omega}{kT}\right)\right]^{-1}$

D. $\left(2\cosh\frac{\hbar\omega}{2kT}\right)^{-1}$

E. $\left(2\sinh\frac{\hbar\omega}{2kT}\right)^{-1}$

Problem is from



Clicker/Poll Question

Suppose we have a two-state system, where the energy of the ground state is 0 and the energy of the excited state is E_0.

What is the expected value of energy for this system at $kT = E_0$?

A. $(1/e)E_0$

B. $(1 - 1/e)E_0$

C. E_0

D. None of the above

E. ???

Try It Yourself

$$\bar{E} = \frac{\sum_i E_i e^{-\beta E_i}}{\sum_i e^{-\beta E_i}} = \frac{1}{Z}\sum_i E_i e^{-\beta E_i} = \frac{1}{Z}\left(-\frac{\partial Z}{\partial \beta}\right) = -\frac{\partial \ln Z}{\partial \beta}$$

Given the single-particle partition function

$$Z_1 = e^{-\beta\mu B} + e^{+\beta\mu B} = 2\cosh(\beta\mu B)$$

A. What is the expected value of energy for this particle?

B. What is the total energy of the system of N dipoles?

C. Redo part (A) with the partition function $Z_N = Z_1^N$, and show that you get the result that you got in part (B).

D. I lied in part (C). Actually, the partition function is $Z_N = \frac{1}{N!}Z_1^N$ if spins are indisguishable. Show that you get the same result as in both parts (B) & (C).

Clicker/Poll Question

For a CO molecule, $\epsilon \approx 2 \times 10^{-4}$ eV. What is the expected energy in rotational mode(s) for an CO molecule at room temperature?

A. $(1/2)kT$

B. kT

C. $2kT$

D. None of the above

E. ???

$$\bar{E} = \frac{\sum_i E_i e^{-\beta E_i}}{\sum_i e^{-\beta E_i}} = \frac{1}{Z}\sum_i E_i e^{-\beta E_i} = \frac{1}{Z}\left(-\frac{\partial Z}{\partial \beta}\right) = -\frac{\partial \ln Z}{\partial \beta}$$

(last slide): $Z_{\text{rot}} \approx \dfrac{kT}{\epsilon}$

Clicker/Poll Question

The preceding proof of the equipartition theorem does not apply to quantum-mechanical systems. Why not?

A. Quantum mechanical systems have discrete energy levels, which are not quadratic.

B. Quantum mechanical systems have relatively big energy-level spacings (compared to kT), which make it a bad approximation to replace the sum with an integral in the proof.

C. The proof only applies to particles with can be considered disguishable. At low enough temperatures where we can’t ignore QM, the fact that we have bosons vs. fermions becomes important.

D. ???

Clicker/Poll Question

Suppose you are able to look down the z-axis and view particles that are either moving towards you or away from you. Of the following, which is the most likely speed that you would measure?

A. 0

B. The most probable speed, moving in the $\pm z$ direction.

C. The rms speed, moving in the $\pm z$ direction.

D. The average speed, moving in the $\pm z$ direction.

E. ???

Clicker/Poll Question

Suppose we lived in 2 spatial dimensions. What would be the form of the distribution function (probability per unit speed inverval)?

A. $\mathscr{D}(v) \propto e^{-\beta m v^2/2}$

B. $\mathscr{D}(v) \propto e^{-\beta m v}$

C. $\mathscr{D}(v) \propto v e^{-\beta m v^2/2}$

D. $\mathscr{D}(v) \propto v e^{-\beta m v}$

E. $\mathscr{D}(v) \propto v^2 e^{-\beta m v}$

Match the following with the required computation (the limits of the integrals are not specified since they depend on the particular quantity)

1. v_{rms}
2. $v_{\text{most probable}}$
3. v_{avg}
4. v_{median}

A. $\int v\mathscr{D}(v)\,dv$

B. $\int v\mathscr{D}(v)\,dv$

C. $\int v^2\mathscr{D}(v)\,dv$

D. $\frac{d}{dv}\mathscr{D}(v)$

Clicker/Poll Question

The previous slide wrote that $S = \frac{U}{T} + k \ln Z$ (for a system of N particles). Is this Z equal to Z_1 or Z_N?

A. Z_1
B. Z_N
C. ???

Clicker/Poll Question

From the thermodynamic identity for F (dF = …), find an expression for the pressure of a system in terms of the partition function.

A. $P = -kT\frac{\partial \ln Z}{\partial V}$

B. $P = +kT\frac{\partial \ln Z}{\partial V}$

C. $P = -kT\frac{\partial Z}{\partial V}$

D. $P = +kT\frac{\partial Z}{\partial V}$

E. None of the above (or not sure)

Clicker/Poll Question

For the nitrogen in this room ($V \sim 100 m^3$), what is the correct order-of-magnitude of the quantum volume V_Q?

A. 10^{-15} m^3.
B. 10^{-23} m^3.
C. 10^{-31} m^3.
D. 10^{-38} m^3.
E. ???

Clicker/Poll Question

There was no degeneracy factor in the previous expression. Why is this?

A. There is no degeneracy in a 3D particle-in-a-box.

B. The degeneracy is in the energy, but since we're summing over triplets of n-values and not energy levels, there's no issue.

C. The degeneracy is a small factor that won't matter for the partition function.

D. ???

Clicker/Poll Question

Which of the following will help in the solution to Problem 6.47?

Problem 6.47. Estimate the temperature at which the translational motion of a nitrogen molecule would freeze out, in a box of width 1 cm.

A. This is the temperature at which $V = V_Q$.

B. This is the temperature where kT is equal to the ground-state energy of a particle-in-a-box.

C. Both of the above are valid and will get the same answer (to the correct order-of-magnitude).

D. Neither of the above is the right approach for this problem.