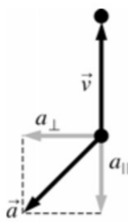
Conceptual Questions

4.1. **(a)** As shown in the figure below, the acceleration $\vec{a}$ can be divided into components perpendicular ($\perp$) and parallel ($||$) to the velocity. $a_{||}$ will slow the particle down since it is in the opposite direction to $\vec{v}$.

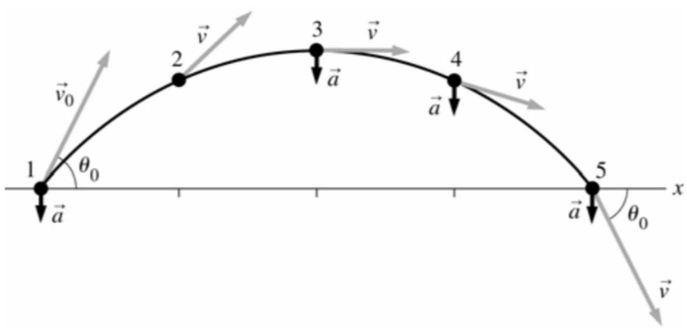
(b) The perpendicular component of $\vec{a}$, $a_{\perp}$, is pointing to the left, and changes the particle direction to the left.



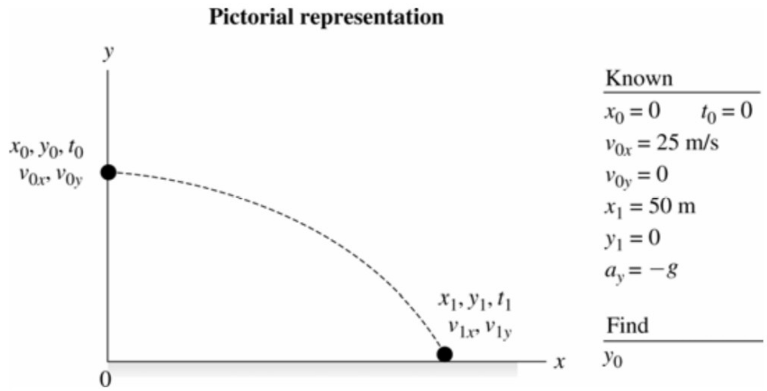
4.4. A typical trajectory of a projectile is shown in the figure below. The acceleration due to gravity always points down. The velocity changes direction from the launch angle $\theta = \theta_0$ above the $+x$-axis to zero at the top of the trajectory, to $\theta = \theta_0$ below the $+x$-axis when it hits the ground.

(a) At no time are $\vec{v}$ and $\vec{a}$ parallel if $\frac{|v_{1y}|}{v_{1x}} = \tan 30°$

(b) At the top of the trajectory $\vec{v}$ and $\vec{a}$ are perpendicular.



4.11. Model: The ball is treated as a particle and the effect of air resistance is ignored.
Visualize:



Solve: Using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$,

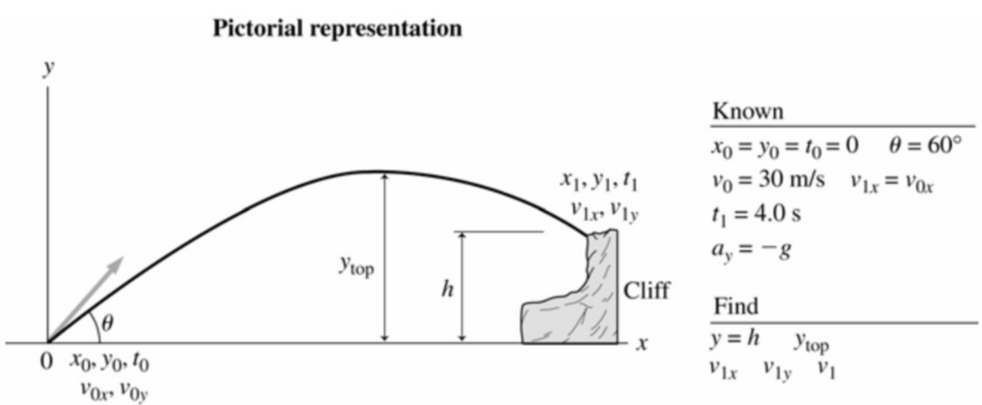
$$50\text{ m} = 0\text{ m} + (25\text{ m/s})(t_1 - 0\text{ s}) + 0\text{ m} \Rightarrow t_1 = 2.0\text{ s}$$

Now, using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$y_1 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)(2.0\text{ s} - 0\text{ s})^2 = -19.6\text{ m}$$

So the ball was thrown from 19.6 m high.
Assess: The minus sign with y_1 indicates that the ball's displacement is in the negative y direction or downward. A magnitude of 19.6 m for the height is reasonable.

4.50. Model: The particle model for the ball and the constant-acceleration equations of motion are assumed.
Visualize:



Solve: **(a)** Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$h = 0\text{ m} + (30\text{ m/s})\sin 60°(4\text{ s} - 0\text{ s}) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(4\text{ s} - 0\text{ s})^2 = 25.5\text{ m}$$

The height of the cliff is 26 m.

(b) Using $(v_y^2)_{top} = v_y^2 + 2a_y(y_{top} - y_0)$,

$$0\ \text{m}^2/\text{s}^2 = (v_0 \sin\theta)^2 + 2(-g)(y_{top}) \Rightarrow y_{top} = \frac{(v_0 \sin\theta)^2}{2\,g} = \frac{[(30\ \text{m/s})\sin 60°]^2}{2(9.8\ \text{m/s}^2)} = 34.4\ \text{m}$$

The maximum height of the ball is 34 m.
(c) The x and y components are

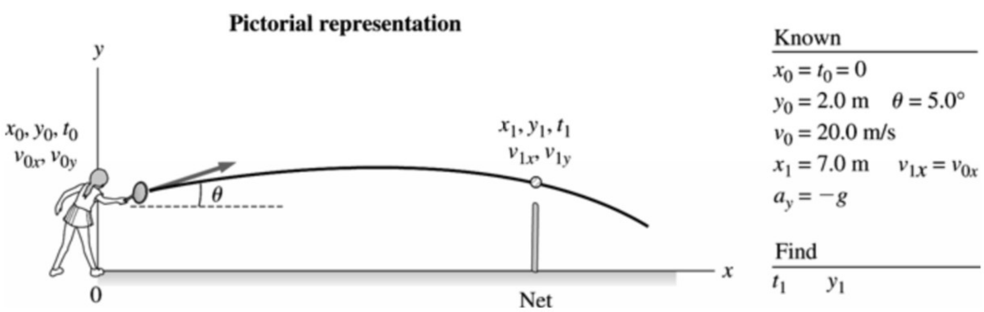
$$v_{1y} = v_{0y} + a_y(t_1 - t_0) = v_0 \sin\theta - gt_1 = (30\ \text{m/s})\sin 60° - (9.8\ \text{m/s}^2) \times (4.0\ \text{s}) = -13.22\ \text{m/s}$$

$$v_{1x} = v_{0y} = v_0 \cos 60° = (30\ \text{m/s})\cos 60° = 15.0\ \text{m/s}$$

$$\Rightarrow v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = 20.0\ \text{m/s}$$

The impact speed is 20 m/s.
Assess: Compared to a maximum height of 34.4 m, a height of 25.5 for the cliff is reasonable.

4.51. Model: The particle model for the ball and the constant-acceleration equations of motion in a plane are assumed.
Visualize:



Solve: The initial velocity is

$$v_{0x} = v_0 \cos 5.0° = (20\ \text{m/s})\cos 5.0° = 19.92\ \text{m/s}$$

$$v_{0y} = v_0 \sin 5.0° = (20\ \text{m/s})\sin 5.0° = 1.743\ \text{m/s}$$

The time it takes for the ball to reach the net is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) \Rightarrow 7.0\ \text{m} = 0\ \text{m} + (19.92\ \text{m/s})(t_1 - 0\ \text{s}) \Rightarrow t = 0.351\ \text{s}$$

The vertical position at $\vec{v} = \vec{v}' + \vec{V}$ is

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$$

$$= (2.0\ \text{m}) + (1.743\ \text{m/s})(0.351\ \text{s} - 0\ \text{s}) + \tfrac{1}{2}(-9.8\ \text{m/s}^2)(0.351\ \text{s} - 0\ \text{s})^2 = 2.01\ \text{m}$$

Thus the ball clears the net by $1.01\ \text{m} \approx 1.0\ \text{m}$.
Assess: The vertical free fall of the ball, with zero initial velocity, in 0.351 s is 0.6 m. The ball will clear by approximately 0.4 m it is thrown horizontally. The initial launch angle of 5° provides some initial vertical velocity and the ball clears by a larger distance. The above result is reasonable.

4.7. Model: Use the particle model for the puck.

Solve: Since the v_x vs t and v_y vs t graphs are straight lines, the puck is undergoing constant acceleration along the x- and y- axes. The components of the puck's acceleration are

$$a_x = \frac{dv_x}{dt} = \frac{\Delta v_x}{\Delta t} = \frac{(10 \text{ m/s} - (-10 \text{ m/s}))}{10 \text{ s} - 0 \text{ s}} = 2.0 \text{ m/s}^2$$

$$a_y = \frac{(10 \text{ m/s} - 0 \text{ m/s})}{(10 \text{ s} - 0 \text{ s})} = 1.0 \text{ m/s}^2$$

The magnitude of the acceleration is $a = \sqrt{a_x^2 + a_y^2} = 2.2 \text{ m/s}^2$.

Assess: The acceleration is constant, so the computations above apply to all times shown, not just at 5 s. The puck turns around at $t = 5$ s in the x direction, and constantly accelerates in the y direction. Traveling 50 m from the starting point in 10 s is reasonable.

4.8. Solve: **(a)** At $t = 0$ s, $x = 0$ m and $y = 0$ m, or $\vec{r} = (0\hat{i} + 0\hat{j})$ m. At $t = 4$ s, $x = 0$ m and $y = 0$ m, or $\vec{r} = (0\hat{i} + 0\hat{j})$ m. In other words, the particle is at the origin at both $t = 0$ s and at $t = 4$ s. From the expressions for x and y,

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \left[\left(\frac{3}{2}t^2 - 4t\right)\hat{i} + (t-2)\hat{j}\right] \text{ m/s}$$

At $t = 0$ s, $\vec{v} = -2\hat{j}$ m/s, $v = 2$ m/s. At $t = 4$ s, $\vec{v} = (8\hat{i} + 2\hat{j})$ m/s, $v = 8.3$ m/s.

(b) At $t = 0$ s, $\vec{v}$ is along $-\hat{j}$, or 90° south of $+x$. At $t = 4$ s,

$$\theta = \tan^{-1}\left(\frac{2 \text{ m/s}}{8 \text{ m/s}}\right) = 14° \text{ north of } +x$$