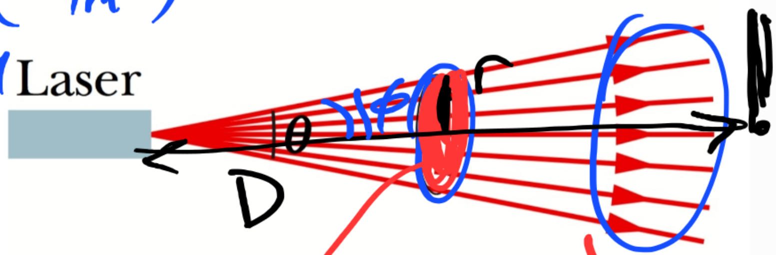
Schedule

Week 4 (today):	Waves/optics, Relativity
5:	Relativity, FACULTY PANEL 5-6
6:	Thermo/Stat. Mech
7:	QM/Atomic I
8:	GRAD PANEL 4-5? ; QM/Atomic II
9:	Miscellaneous (presentations)

22. A helium-neon laser, radiating at 632.8 nm, has a power output of 3 mW. The beam diverges (spreads) at total angle $\theta = 0.25$ mrad (this is the *total* angle, not the half-angle). What is the amplitude of the electric field due to the laser light 75 m from the laser?

A) 100 V/m
B) 10 V/m
C) 1 V/m
D) 0.1 V/m
E) 0.01 V/m



Power (W) ... P, I ... intensity (W/m²)

$I = \frac{P}{A} \stackrel{?}{=} \frac{P}{4\pi r^2}$

$I = \frac{P}{\pi r^2}$

$\tan\phi \approx \phi = \frac{\theta}{2} = \frac{r}{D}$

$r = D\frac{\theta}{2}$

$= (75\,m)(0.125\times 10^{-3})$

$\approx 10^{-2}\,m$

$I = \frac{3\,mW}{\pi (10^{-4}\,m^2)}$

$\sim 10\ \frac{W}{m^2}$

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

$I = \langle \vec{S} \rangle = \frac{E_0 B_0}{2\mu_0}$

$10 = \frac{E_0^2}{2c\mu_0}$

$E_0 = \sqrt{20\,c\,\mu_0}$ ($3\times10^8 \cdot 4\pi\times10^{-7}$)

$\approx 5(400)^{1/2}$

$\approx 5\cdot 20$

≈ 100 V/m

27. What is the minimum separation of two objects on Mars in order to resolve them by an observer on Earth with the naked eye? The distance to Mars is 8.0×10^7 km, the diameter of the pupil is 5.0 mm, and the wavelength of light can be taken to be 550 nm.

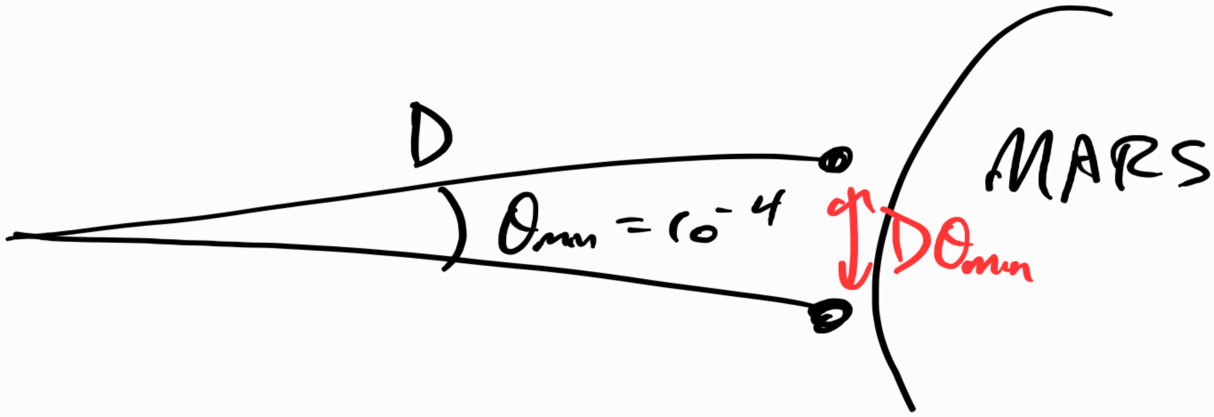
A) 10^4 km
B) 10^3 km
C) 10^5 km
D) 10^2 km
E) 10^6 km

Rayleigh Diffraction Limit

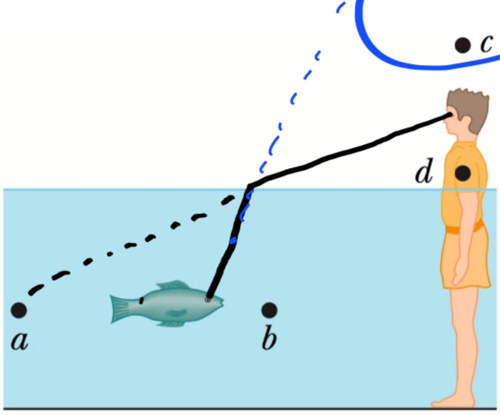
$$\theta_{min} \approx \frac{1.22\lambda}{D} \sim \frac{\lambda}{D}$$

5×10^{-3}

$$\theta_{min} \sim \frac{500 \times 10^{-9}}{5 \times 10^{-3}} \sim 10^{-4}$$



24. In the figure below, a person looks at a fish and a fish looks at a person. (i) Does the person see the fish's image closer to point a or point b? (ii) Does the fish see the person's image closer to point c or point d?



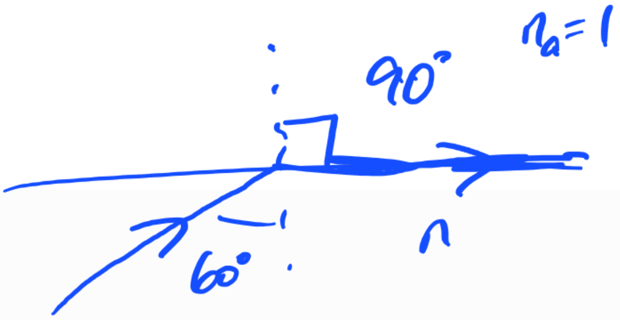
A) (i) Point a ; (ii) Point c
B) (i) Point a ; (ii) Point d
C) (i) Point b ; (ii) Point c
D) (i) Point b ; (ii) Point d

21. Which of the following statements is TRUE regarding mechanical waves?

A) The denser the material, the higher the speed of sound in that material.
B) Sound waves are the most common example of transverse mechanical waves.
C) Upon reflection, a wave on a string can have its waveform inverted.
D) The doppler effect says that it is the frequency only that changes, and not the wavelength of sound waves, when a source moves towards a receiver.
E) When the frequency of waves doubles, the speed of those waves also doubles.

23. Total internal reflection occurs at an interface between a plastic and air at incidence angles larger than 60 degrees. What is the refractive index of the plastic?

A) 0.707
B) 1.41
C) 1.5
D) 1.73
E) 1.15



$n_a \sin 90° = n \sin 60°$

$(1)(1) = \frac{\sqrt{3}}{2} n$

$n = \frac{2}{\sqrt{3}} = \frac{2}{1.73}$

25. You're designing your very own bathroom mirror. You'd like the mirror to have magnification 5 when your face is 20 cm away from the mirror. What should be the radius of curvature of the mirror? Should the mirror be concave or convex?

A) Concave mirror, radius of curvature 50 cm.
B) Concave mirror, radius of curvature 12.5 cm.
C) Convex mirror, radius of curvature 50 cm.
D) Convex mirror, radius of curvature 12.5 cm.
E) None of the above.

$m = +5$ requires a concave (converging) mirror.

$m = 5 = \frac{-o'}{o}$

$\frac{1}{f} = \frac{1}{o} + \frac{1}{o'}$

$\frac{o}{f} = 1 + \frac{o}{o'} = 1 - \frac{1}{5} = \frac{4}{5}$

$f = \frac{5}{4} o = \frac{5}{4} [20\text{cm}] = 25\text{ cm}$

$R = 2f = 50\text{ cm}$

Relativity Facts

* Lorentz Transformations (compare to Galilean)
* $\gamma = \frac{1}{\sqrt{1-\beta^2}} \geq 1 \quad \left(\beta = \frac{v}{c}\right)$
* length contraction: $L' = \frac{L_0}{\gamma}$ (proper length)

  time dilation: $\Delta t = \gamma \Delta \tau$ (Proper time)

* Velocity Addition

$$V' = \frac{u+v}{1+\frac{uv}{c^2}}$$

if these are opp.

* Relativistic Doppler

$$f = f_0 \left(\frac{1 \pm \beta}{1 \mp \beta} \right)^{1/2}$$

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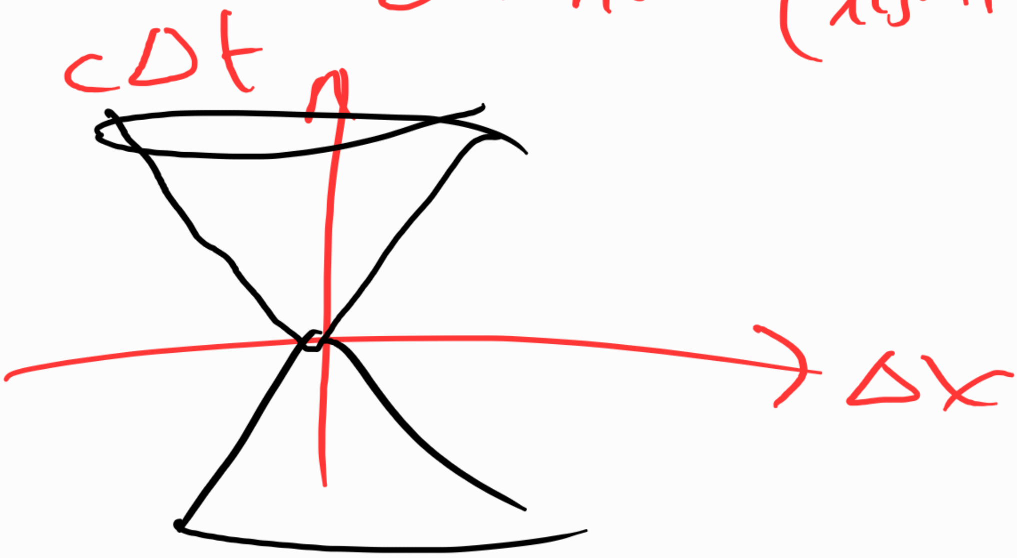
* $\Delta s^2 = c^2 \Delta t^2 - \Delta x^2$ (interval)

+ : timelike sep

− : Spacelike - sep

0 : null (lightlike)



$$\begin{pmatrix} \Delta x' \\ c\Delta t' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \Delta x \\ c\Delta t \end{pmatrix}$$

If S' is moving in $\beta\hat{x}$ as measured by S.

1. System $\overline{S}$ travels with constant velocity $v \neq 0$ in the $\hat{\mathbf{x}}$-direction with respect to system S. If two events, separated by a distance $x \neq 0$, occur simultaneously at time t in S, do they occur simultaneously in $\overline{S}$?

$\Delta t = 0$

(A) Yes, always

(B) No, never

(C) Only if $x < vt$

(D) Only if $x > vt$

(E) Only if $x < ct$

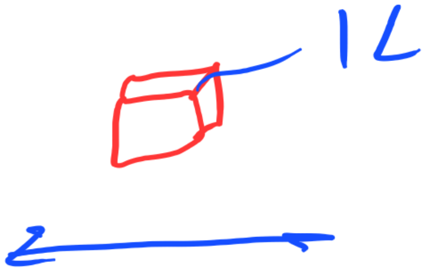
$$\begin{pmatrix} \Delta x' \\ c\Delta t' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \Delta x \\ 0 \end{pmatrix}$$

$$c\Delta t' = -\beta\gamma \underbrace{\Delta x}_{\text{nonzero}}$$

SPACELIKE - SEP

9. Spaceship 1, carrying a meter stick, flies past Spaceship 2, carrying a 1 liter container. The occupants of Spaceship 2 measure the meter stick on Spaceship 1 to be 0.5 m long. What volume do the occupants of Spaceship 1 measure for the container on Spaceship 2? Both spaceships travel along parallel trajectories and all dimensions should be measured parallel to the axis of their trajectories.

(A) 0.125 L
(B) 0.25 L
(C) 0.5 L
(D) 1 L
(E) 2 L



turns out, problem means IN ENT. frame...

7. The USS *Enterprise*, moving at speed 0.5c with respect to a nearby planet, fires a photon torpedo of speed c at a Romulan warship, initially 6000 km away, which is retreating away from the *Enterprise* at constant velocity. According to the *Enterprise's* clock, the torpedo made contact with the warship 0.1 seconds after firing. How fast was the warship traveling, in the frame of the planet?

as measured in which frame?

1 ls = 300000 km

(A) $\frac{13}{28}c$

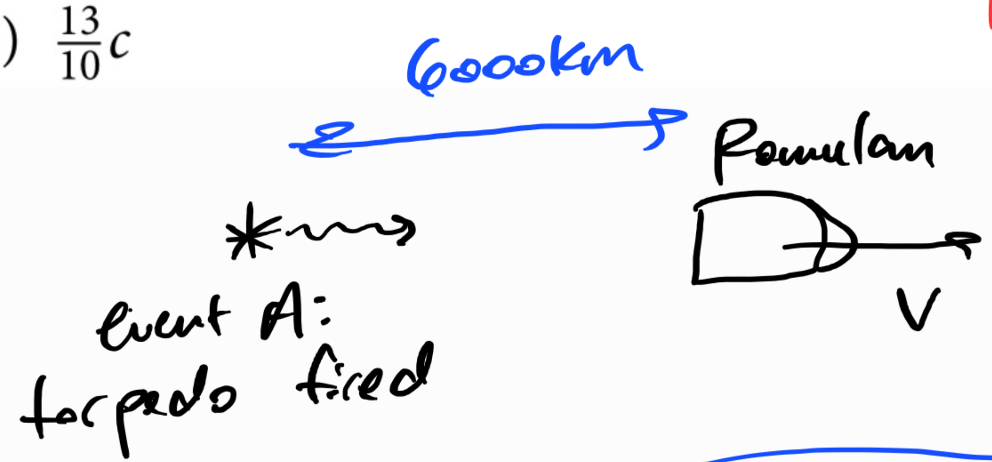
(B) $\frac{13}{16}c$

(C) $\frac{13}{14}c$

(D) c

(E) $\frac{13}{10}c$

How fast does the light travel in the frame of the planet?



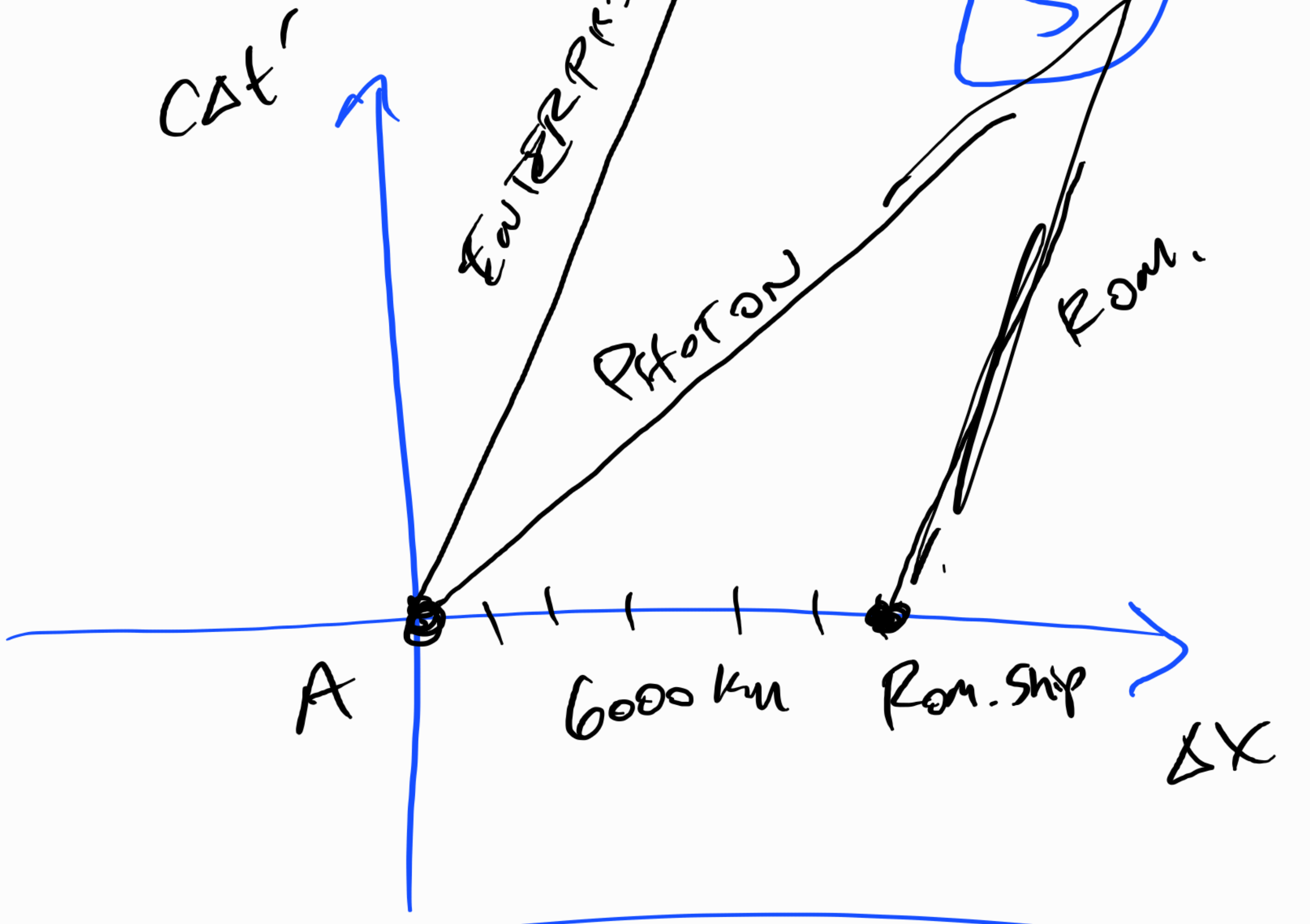
S: planet

S': Enterprise

$$0 = (c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$? - (?) = (0.1\,\text{ls})^2 - ?$$

Knowns: $c\Delta t'$



Added after lec:

$$\gamma \equiv \frac{1}{\sqrt{1-0.5^2}} = \frac{2}{\sqrt{3}}$$

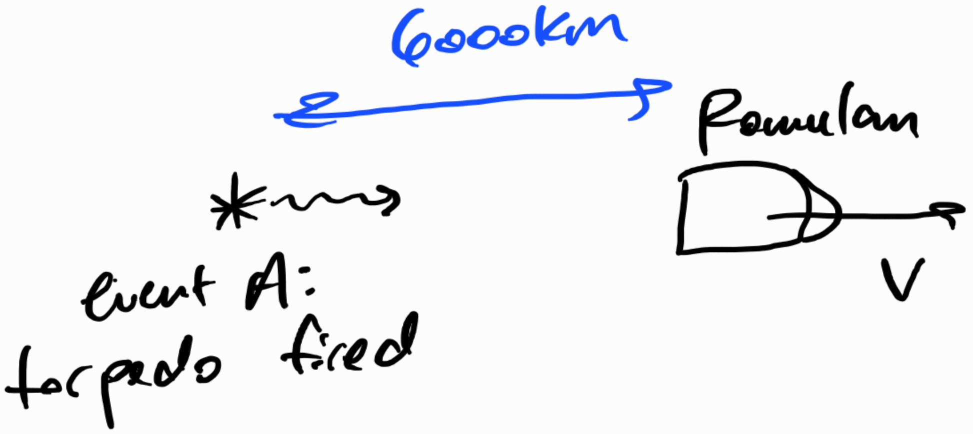
Assuming 6000 km in planet frame

In the enterprise frame, $\Delta x' = c\Delta t' = 0.1\,ls$.

$$\begin{pmatrix} \Delta x \\ c\Delta t \end{pmatrix} = \begin{pmatrix} \gamma & +\beta\gamma \\ +\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} 0.1\text{ l.s.} \\ 0.1\text{ l.s.} \end{pmatrix}$$

$$= \begin{pmatrix} 2/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 2/\sqrt{3} \end{pmatrix} \begin{pmatrix} 0.1\text{ l.s.} \\ 0.1\text{ l.s.} \end{pmatrix}$$

$\Delta x = c\Delta t = \sqrt{3}$ l.s.



$$6000\text{km} + V_{Rom}\Delta t = \Delta x$$

$$V_{rom} = c - \frac{6000\text{km}}{\Delta t}$$

1 l.s. = 300,000km

6000 km = $\frac{1}{50}$ l.s.

$$= c\left[1 - \frac{6000\text{ km}}{\sqrt{3}\text{ l.s.}}\right]$$

$$= c\left[1 - \frac{1/50}{\sqrt{3}}\right]$$

$50\sqrt{3}$

$\frac{1}{2}(170)$

$= 85$

$$\approx c\left[1 - \frac{1}{85}\right]$$

UGH. Not an answer choice...

maybe 6000km is not distance in planet frame, but in E's frame

SOL 2 (enterprise)

(taking distance = 6000km in the ENTERPRISE frame)

∴

$$\Delta x' = c\Delta t' = 6000\,km + V'_{rom}\,\Delta t'$$

$$0.1\ l.s. = 0.02\ l.s + V'_{rom}(0.1\,sec)$$

Speed of romulan ship in enterprise frame → $V'_{rom} = 0.8c$

Velocity addition

$$ANS = \frac{0.8+0.5}{1+(0.8)(0.5)} = \frac{1.3}{1+0.4} = \frac{13}{14}$$

(C)

Sorry the question wasn't more particular!!