QM: FACTS (?) $(\hat{K}+\hat{V})\psi = E\psi$

General

* Time evolution follows T.D.S.E.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

* T.I.S.E. follows from $\Psi(x,t) = \Phi(t)\psi(x)$

$$\Phi(t) = e^{-i\omega t} = e^{-iE_n t/\hbar}$$

* ~~$\langle H \rangle$ is an energy eigenvalue~~

  True only if ψ is an energy eigenstate, but not in general.

* S.E. generates an ∞ set of orthogonal solutions

* $\psi(x)$ and $\psi'(x)$ are continuous.

  (if $V(x)$ is finite)

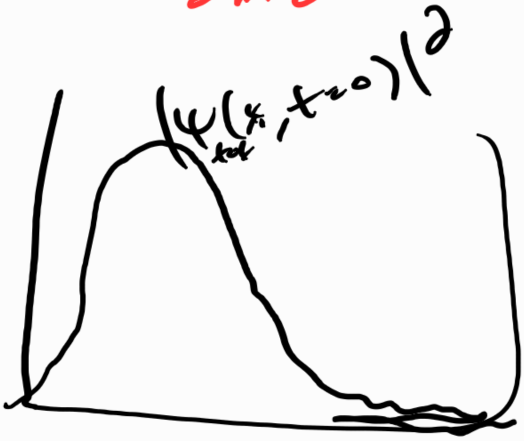
* Photoelectric effect: $eV_s = hf - \phi = K_{max}$

* If ψ_1 is an allowed wavefunction, and ψ_2; then $c_1\psi_1 + c_2\psi_2$ is okay

Specific Problems

H.O. $E_n = (n+\frac{1}{2})\hbar\omega$, $n = 0, 1, 2 \ldots$

PIB. $$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}, \quad n = 1, 2, \ldots$$



$$\psi_{tot} = \frac{1}{\sqrt{2}}\left[\psi_1 + \psi_2\right]$$

Transmission / Reflection

$k_2 < k_1$

$$\psi(x) = Ae^{ik_1x} + Be^{-ik_1x}$$

$$\psi(x) = Ce^{ik_2x}$$

$$k = \frac{\sqrt{2m(E-V(x))}}{\hbar}$$

$$T = \frac{4k_1k_2}{(k_1+k_2)^2}$$

$$R = 1 - T$$

University of California, San Diego
GRE Prep – Spring 2022
Practice Quantum Mechanics Problems
May 2, 2022

NOTE: Problems 1-3 are all based on the 1D particle-in-a-box setup below:

Suppose you have a particle in a 1D infinite well, where the potential is zero if $|x| < L/2$ and $+\infty$ otherwise. All distinct energy eigenstates can be labeled with position-space energy eigenstates $\psi_n(x)$, where n is a positive integer and $\psi_n(x) = \sqrt{2/L}\sin(n\pi x/L)$ is real. Suppose that at $t = 0$ you start with the state

$$\psi(x, t = 0) = N\Big(\psi_1(x) + 2\psi_2(x)\Big)$$

In the following, the "left" side of the box refers to the region $-L/2 < x < 0$, the "right" side of the box refers to the region $0 < x < L/2$, and the "middle" of the box is the point $x = 0$.

---

1. (This problem is based on the initial setup.) Which of the following is a possible normalization factor, N?

A) $i/3$
B) $1/\sqrt{3}$
C) $1/5$
D) $i/\sqrt{5}$
E) None of the above

---

2. (This problem is based on the initial setup.) Which of the following best describes the expected value of the position of the particle over time, $\langle x \rangle$?

A) $\langle x \rangle$ is fixed over time and in the middle of the box.
B) $\langle x \rangle$ is fixed over time and somewhere on the left side of the box.
C) $\langle x \rangle$ is fixed over time and somewhere on the right side of the box.
D) $\langle x \rangle$ is not fixed over time and switches between the left and right sides of the box.
E) None of the above.

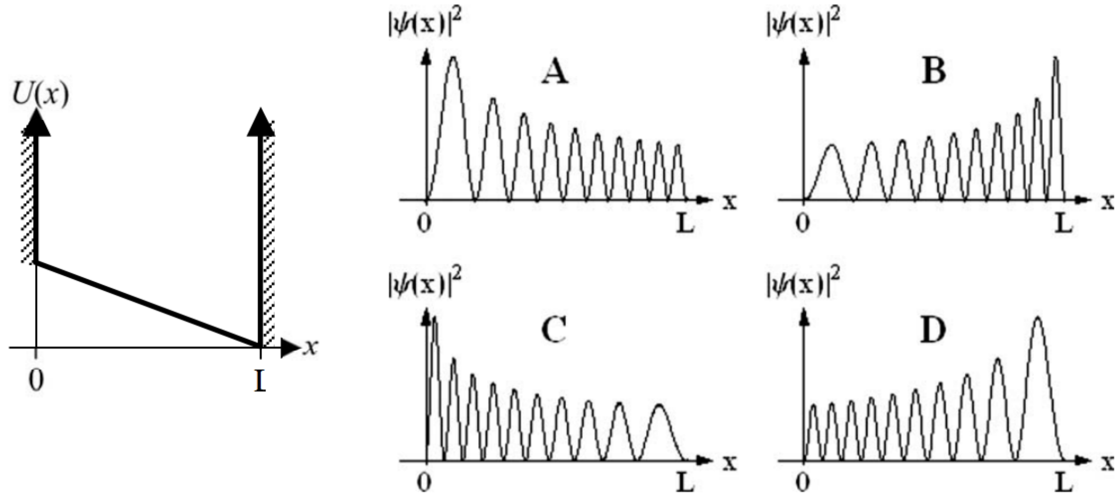
---

3. (This problem is based on the initial setup.) Which of the following best describes the expected value of the energy of the particle over time, $\left\langle \hat{H} \right\rangle$ (where $\hat{H}$ is the Hamiltonian)?

A) $\left\langle \hat{H} \right\rangle$ is fixed over time and equal to the average of E_1 and E_2.
B) $\left\langle \hat{H} \right\rangle$ is fixed over time and less than the average of E_1 and E_2.
C) $\left\langle \hat{H} \right\rangle$ is fixed over time and greater than the average of E_1 and E_2.
D) $\left\langle \hat{H} \right\rangle$ is not fixed over time and switches between E_1 and E_2.
E) None of the above.

---

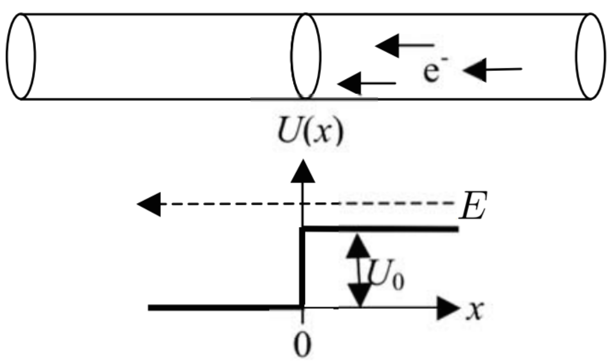
4. The figure below shows a potential energy function $U(x)$, where the potential energy is infinite if x is less than 0 or greater than L, and has a slanted bottom in between 0 and L, so that the potential well is deeper on the right than on the left. Which of the plots of $|\psi(x)|^2$ vs. x is most likely to correspond to a stationary state of this potential well?



E. They are all possible stationary states.

---

5. A beam of electrons that all have the same energy E are traveling through a conducting wire. At $x = 0$, the wire becomes a different kind of metal so that the potential energy of the electrons decreases from U_0 to 0. If $E > U_0$, which statement most accurately describes the transmission and reflection of electrons?



A) All the electrons are transmitted because they all have $E > U_0$.
B) Some of the electrons are transmitted and some are reflected because they actually have a range of energies.
C) Some of the electrons are transmitted and some are reflected because they behave as waves.
D) All of the electrons are transmitted because the potential energy drops, so there is no reason for them to be reflected.
E) The situation described in the problem statement is impossible because E can't be greater than U_0 (a consequence of the Schrodinger Equation).

---

6. You see a free electron and a free neutron moving by you at the same speed. How do their wavelengths compare? In the following, λ_n refers to the wavelength of the neutron, and λ_e refers to the wavelength of the electron.

A) $\lambda_n > \lambda_e$
B) $\lambda_n < \lambda_e$
C) $\lambda_n = \lambda_e$
D) Impossible to say: depends on the speed.
E) Impossible to say: depends on whether the two are moving in the same or opposite directions.

---

7. Suppose a particle is trapped in a "half" harmonic oscillator. That is, the potential can be described by $V(x) = (1/2)m\omega^2x^2$ if $x > 0$ and $+\infty$ if $x \leq 0$. If the particle jumps down from an excited state to a lower-energy state, releasing a massless particle X, which of the following could possibly be the energy of the emitted particle X? Ignore angular momentum.

A) $(1/2)\hbar\omega$
B) $\hbar\omega$
C) $(3/2)\hbar\omega$
D) $2\hbar\omega$
E) None of the above.

---

8. Which of the following is equal to the commutator $[\hat{x}\hat{p}, \hat{p}\hat{x}]$?

A) $-2i\hbar$
B) $-i\hbar$
C) 0
D) $+i\hbar$
E) $+2i\hbar$

---

9. By shining UV light of wavelength 248 nm on a metal, the stopping voltage is found to be 2.0 eV. What is the stopping voltage if light of wavelength 620 nm is shined on the metal?

A) 1.0 eV
B) 0.4 eV
C) 2.5 eV
D) 2.0 eV
E) N/A (no electrons are ejected from the metal for this wavelength.)

---

10. Approximately how many photons per second does a 100-Watt sodium lamp ($\lambda = 589$ nm) radiate?

A) 3×10^{20}
B) 3×10^{21}
C) 3×10^{22}
D) 1×10^{19}
E) 1×10^{20}

---

11. Suppose you have the following operator:

$$\hat{A} = \begin{pmatrix} 1 & 1-i \\ 1+i & 0 \end{pmatrix}$$

Which of the following statements are TRUE?

I. The operator could possibly represent an observable for a quantum-mechanical system.
II. Both of the eigenvalues of the operator are positive.
III. The operator satisfies the equation $\hat{A}^2 - \hat{A} = \mathbb{1}$

A) Statement I only.
B) Statement II only.
C) Statement III only.
D) Statements I and III only.
E) Statements I, II, and III.

---

12. An electron is in an orbital of hydrogen. It is known that the angular momentum has magnitude $\sqrt{6}\hbar$, and the z-component of the angular momentum is $-\hbar$. What orbital is the electron in?

A) 6p
B) 5d
C) 4f
D) 3d
E) Cannot be determined from the information given.

---

13. If the location of a grain of sand (mass 1mg) is measured to within a nanometer, then what is the minimum uncertainty in its speed?

A) 10^{-19} m/s
B) 10^{-25} m/s
C) 10^{-5} m/s
D) 10^{19} m/s
E) 10^{25} m/s

---

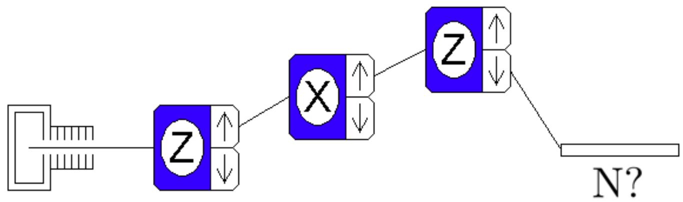
14. Which of the following predictions of the Bohr model is NOT true?

A) Electrons in a given orbital do not radiate as they move around the nucleus in that orbit.
B) The orbital angular momentum of the electron around the nucleus is a positive integer multiple of $\hbar$.
C) The energy levels of a hydrogen atom are proportional to $1/n^2$, where n is the principal quantum number.
D) The emission spectrum of hydrogen can be explained by energy differences in energy levels accessible to the electron.
E) N/A – all of the above are true statements about the hydrogen atom (not taking into account any fine structure).

---

15. A beam of 1000 unpolarized spin-1/2 particles are passed through three Stern-Gerlach (SG) apparatuses as shown below. The devices are oriented along either the x or z direction (as indicated), and either the spin-up or spin-down beam is kept after each (indicated with the line going on past the device). What is the expected number of particles that will have passed through all three SG devices?

A) 0
B) 125
C) 250
D) 500
E) 1000



---

16. A spin-1/2 system is in the state described by the spinor

$$\chi = \begin{pmatrix} i \\ -i \end{pmatrix}$$

The basis for the spinors is the z-basis. The above state is an eigenstate of which of the following? Recall σ_i is the Pauli spin operator along the i'th direction, where i is x, y, or z:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

A) σ_z
B) σ_y
C) σ_x
D) $\sigma_x \sigma_y$
E) $\sigma_x \sigma_z$

---

17. Which of the following best describes the number and types of nodes of a 4p orbital?

A) There are 2 total nodes: 1 angular and 1 radial.
B) There are 2 total nodes: 2 angular and 0 radial.
C) There are 3 total nodes: 2 angular and 1 radial.
D) There are 3 total nodes: 1 angular and 2 radial.
E) There is 1 total node: 0 angular and 1 radial.

---

18. What is the ground-state configuration of a neutral Oxygen atom? (Oxygen has 16 total nucleons: 8 protons and 8 neutrons)

A) $1s^2 2s^2 2p^6 3s^2 3p^6$
B) $1s^2 2s^2 2p^6 3s^2 3p^4$
C) $1s^2 2s^2 2p^8$
D) $1s^2 2s^2 2p^6$
E) $1s^2 2s^2 2p^4$

End of Multiple Choice Questions

QUANTUM MECHANICS (12% of exam):

(a) Lower-Division Basics:

  (i) Photoelectric effect
  (ii) Compton Scattering
  (iii) de Broglie
  (iv) Heisenberg Uncertainty

(b) Common Potentials / Situations:

  (i) Particle-in-a-box (infinite or finite)
  (ii) Harmonic Oscillator (1D, 3D including degeneracy)
  (iii) Free Particle
  (iv) Dirac Delta Function

(c) Position-Space Wavefunctions

  (i) Probability Density
  (ii) Expectation Values of operators given $\psi(x)$
  (iii) $\hat{x}$ and $\hat{p}$ in position-space representation

(d) General Wavefunctions, Formalism, Matrix Mechanics / Linear Algebra

  (i) Normalization
  (ii) Eigenvalues / Eigenvectors
  (iii) Measurement, Collapse of Wavefunction
  (iv) Commutation Relations
  (v) Properties of Hermitian Operators
  (vi) Schrodinger Equation and time-evolution

ATOMIC PHYSICS (10% of exam)

(a) Bohr Model and Hydrogen

  (i) Assumptions of Model
  (ii) Energy Levels of Hydrogen; Lyman, Balmer series
  (iii) Positronium

(b) Atomic Structure / Chemistry

  (i) Electron Configuration
  (ii) Shape of Orbitals

(c) Spin and Spinors

  (i) Pauli Matrices, Eigenvalues / Eigenvectors
  (ii) Stern-Gerlach Experiment (main result)