Expansion of the Universe

Physics 198: Special Topics

$$1 + z = \frac{a(t_0)}{a(t_e)} = \frac{1}{a(t_e)}.$$

1. Cosmological Redshift

- Caused by the combination of the expansion of the universe and the constant speed of light
- The distance of the source, as well as the age of the universe and the cosmological sizes at the time of the emission of the incident light, can be approximated

$$z = \frac{H_0}{c} r,$$

2. Hubble's Law

- Hubble related the distance of galaxies to their redshift with the above relations
- Hubble time is shown below. This shows the time that has elapsed since galaxies were "in contact" based on no forces acting on the galaxies

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1},$$

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1). \end{cases}$$

3. Space and Spacetime

- The curvature of space (positively, flat, or negatively curved) also strongly affects the volume, and therefore expansion, of the universe
- Robertson-Walker metric - shows the spacetime separation of events, taking curvature of space into account

$$ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + S_\kappa(r)^2 d\Omega^2\right],$$

$$H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{\kappa c^2}{R_0^2},$$

4. Equations of State

- Above - Friedmann Equation
  - H_0 - current rate of expansion
  - ε_0 - current energy density
  - κ - current curvature
- Left - Acceleration Equation
  - ε - energy density
  - P - pressure associated with material filling the universe
- Right - Fluid Equation
  - ε - energy density
  - P - pressure associated with material filling the universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P).$$

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0.$$

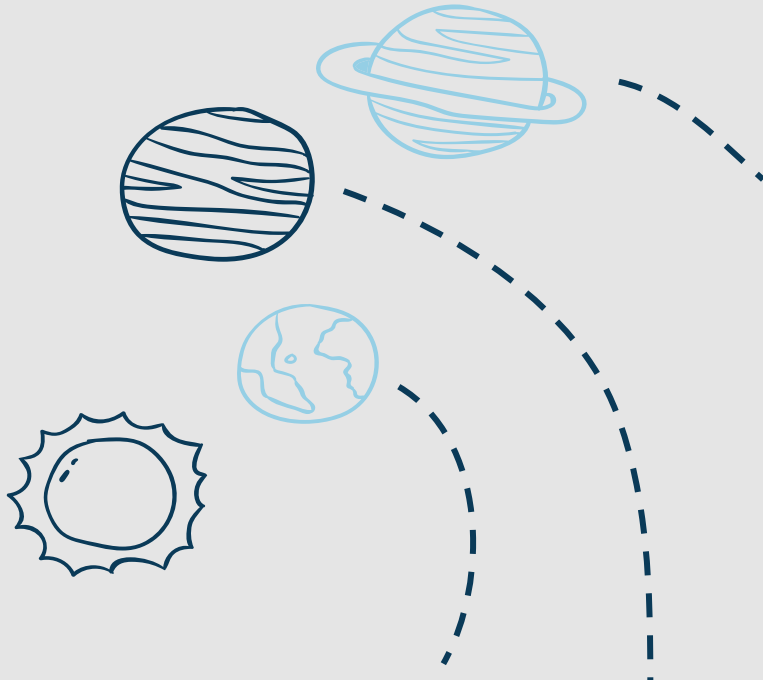
Cosmological Redshift

$$z \equiv \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}} \frac{\lambda_e}{a(t_e)} = \frac{\lambda_0}{a(t_0)}$$

$$1 + z = \frac{a(t_0)}{a(t_e)} = \frac{1}{a(t_e)}.$$

If a source in the night sky appears to have redshift z = 8, what was the scale factor of acceleration at the time of emission?

A. 0.11
B. 9.01
C. 0.51
D. 0.32
E. 0.25



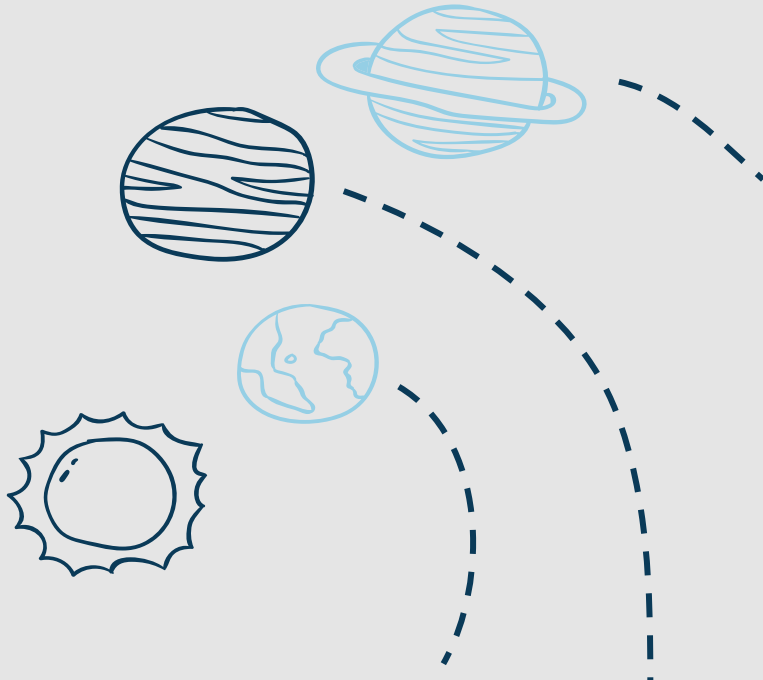
Space and Spacetime

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1). \end{cases}$$

$$ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + S_\kappa(r)^2 d\Omega^2\right]$$

Consider an equilateral triangle, with sides of length L, draw on a two-dimensional surface of uniform curvature. On what surfaces with radius of curvature R, k = +1, k = 0, and/or k = -1, can you draw an equilateral triangle of arbitrarily large area A?

A. k = 0
B. k = +1 and k = 0
C. k = +1 and k = -1
D. k = 0 and k = -1
E. All of the above



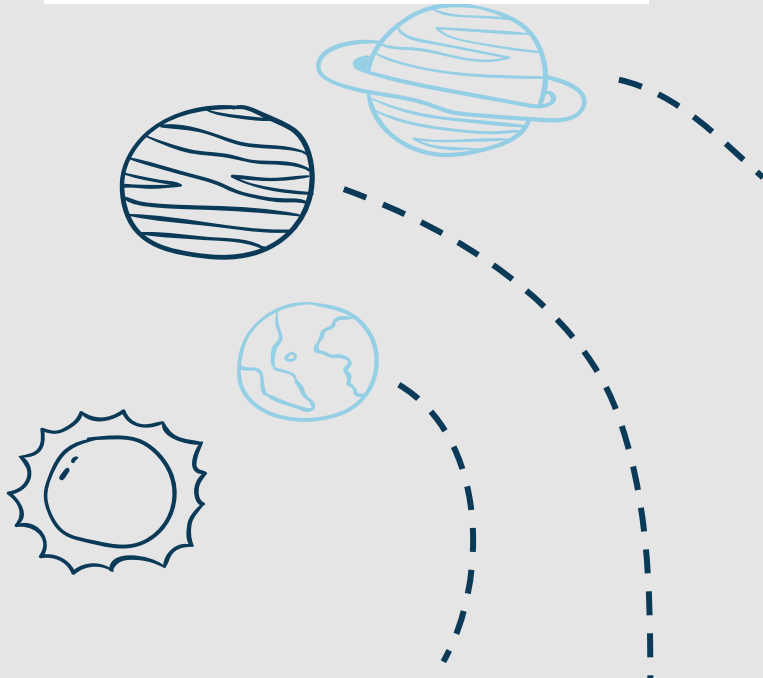
Equations of State

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3}$$

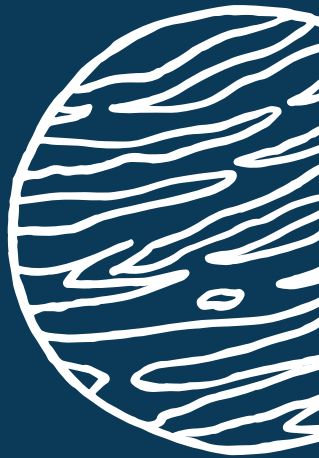
$$P_\Lambda = -\varepsilon_\Lambda = -\frac{c^2}{8\pi G}\Lambda$$

Einstein added a factor of lambda to the acceleration equation of state to try to match his prediction that the universe would be static. It was then rewritten as in the top right, with pressure as the equation below that. What is the value of lambda?

A. $4\pi G\rho/3$
B. $6\pi G/8c^2$
C. $4\pi G\rho$
D. $6\pi G\rho/8$
E. $3\pi G/4c^2$



Thanks!



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