Introduction to Condensed Matter Physics

- for GRE preparation, by Thomas, Eric and Rohan

Resources

- Wikipedia: https://en.wikipedia.org/wiki/Solid-state_physics
  - https://en.wikipedia.org/wiki/Condensed_matter_physics

So You Want to Learn Physics...

- https://www.susanrigetti.com/physics
- Simon, The Oxford Solid State Basics: 9780199680771
- Lubensky, Principles of Condensed Matter Physics: 978-0521794503
- **Conquering the Physics GRE**

What we're getting into: Sample questions

The Meissner effect refers to the expulsion of magnetic fields from a superconductor. The exponential decrease in field strength inside the conductor can be modeled by giving the photon an effective

(A) positive electric charge
(B) negative electric charge
(C) mass
(D) spin
(E) color charge

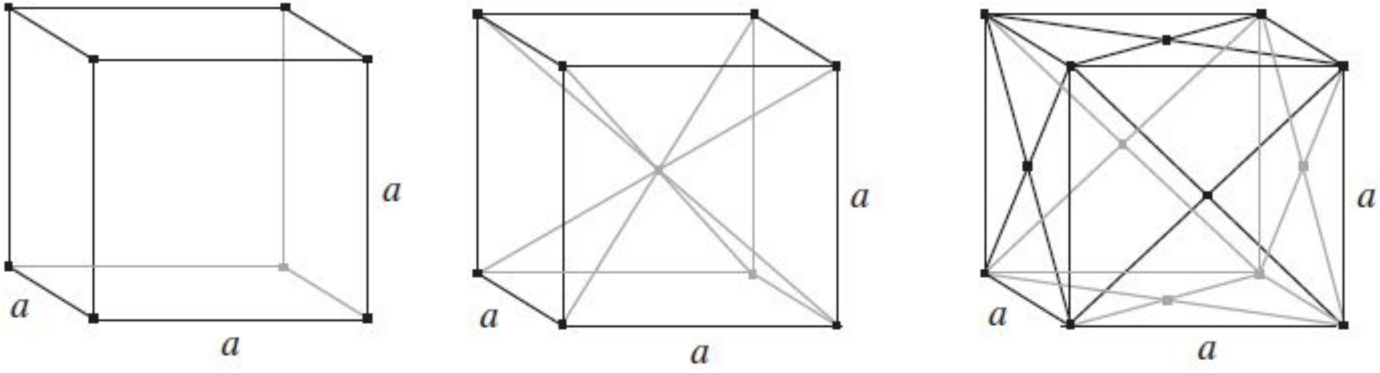
The electrical conductivity of a relatively pure semiconductor increases with increasing temperature primarily because:

(A) The scattering of the charge carriers decreases.
(B) The density of the charge carriers increases.
(C) The density of the material decreases due to volume expansion.
(D) The electric field penetrates further into the material.

Basics: keywords

- Phases, phase transitions.
- Magnetism, ferromagnetism.
- Emergent properties.
- Cold stuff.
- Superconductors.

The unit cell



(a) Simple cubic (b) Body-centered cubic (BCC) (c) Face-centered cubic (FCC)

All crystals are 3D copies of unit cells

Fixing the volume of one unit cell

- Atomic distances:
- SCC: a
- BCC: a sqrt(3)/2
- FCC: a sqrt(2)/2
- Common GRE question: apply these formulae

Fourier Transforming to momentum space

- The simple cubic is its own dual lattice. For a lattice of side length a, the dual lattice has side length $2\pi/a$ (the 2π comes from the Fourier transform, and the $1/a$ since the lattice vectors are supposed to have units of wavenumber).
- The body-centered cubic and face-centered cubic lattices are dual to one another.
- The dual to a hexagonal lattice is another hexagonal lattice, but rotated through a $30°$ angle.

The primitive unit cell of the reciprocal lattice is so important that it is given its own name: the *(first) Brillouin zone*.

Transform to momentum space, but skip Fourier Transform

For any lattice **R**, with lattice sites **a**, you can use these to find the reciprocal lattice **G** with lattice sites **b**.

$$e^{-\vec{G}\cdot\vec{R}} = 1$$

$$\vec{G} = \vec{k} \cdot \vec{b}$$

$$\vec{R} = \vec{n} \cdot \vec{a}$$

Electron Theory of Metals

$$k_F \propto n^{1/3}$$
$$E_F \propto n^{2/3}$$

Valence electrons are delocalized due to QM nature of fermions.

Roaming electrons exist on the surface of the ***Fermi Sphere***, with ***Fermi wavevectors*** in phase space.

$|\mathbf{k}| = k_F$, the *Fermi wavevector*.

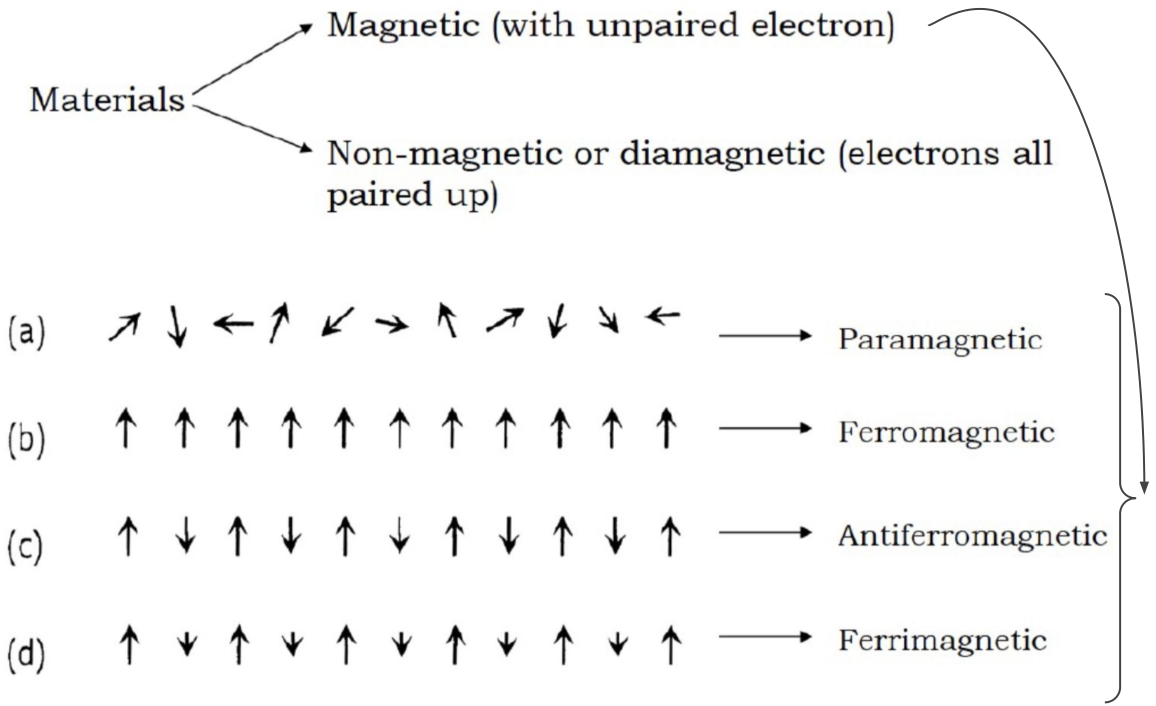
$$k_F = (3\pi^2 n)^{1/3}$$

Using k_F we can define the ***Fermi Energy*** as the kinetic energy of roaming electrons at T = 0K.

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$\rho(E) = \frac{V\sqrt{2}}{\pi^2\hbar^3} m^{3/2}\sqrt{E} = \frac{3}{2}\frac{N}{E_F} \Rightarrow N_C \approx \rho(E_F)(k_B T) \sim N\frac{k_B T}{E_F}$$

Magnetism



Materials
Magnetic (with unpaired electron)
Non-magnetic or diamagnetic (electrons all paired up)
(a)
Paramagnetic
(b)
Ferromagnetic
(c)
Antiferromagnetic
(d)
Ferrimagnetic

Magnetism

Paramagnets follow Curie law:

As the temperature increases, paramagnets become less susceptible to magnetization (Chi decreases)

$$\chi = \frac{C}{T}$$

C: Curie constant

T: temperature

Magnetism

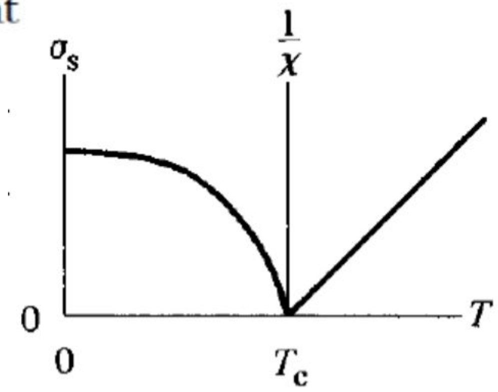
Anti-/Ferromagnets follow Curie-Weiss law:

$$\chi = \frac{C}{T - \theta}$$

θ: Weiss constant

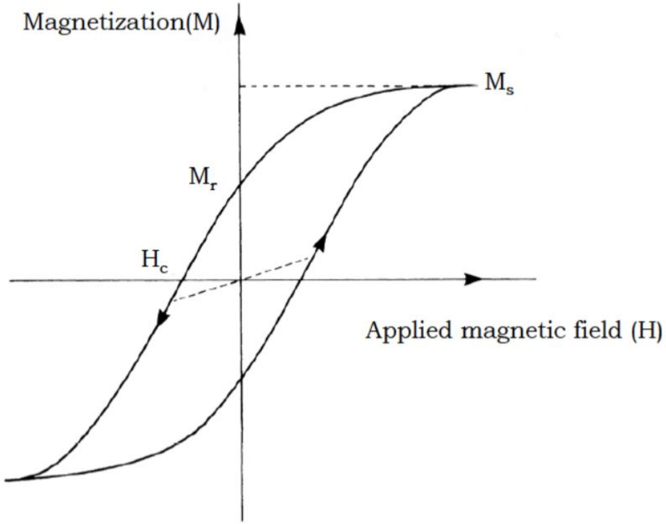
There is a Temperature (Weiss constant) where there is a spontaneous interaction between neighboring spins.

Below Weiss constant the sample is either anti-/ferro, and above Weiss constant the sample is paramagnetic.



Magnetism

Hysteresis Loop



M_s: Saturation magnetization
M_r : Remanent magnetization
H_c: Coercive magnetic field

Magnetically *Hard*

- High M_r and H_c
- Magnetization remains after the field switch off (permanent magnet)

Magnetically *Soft*

- Low M_r and H_c (small area)
- Easily demagnetized

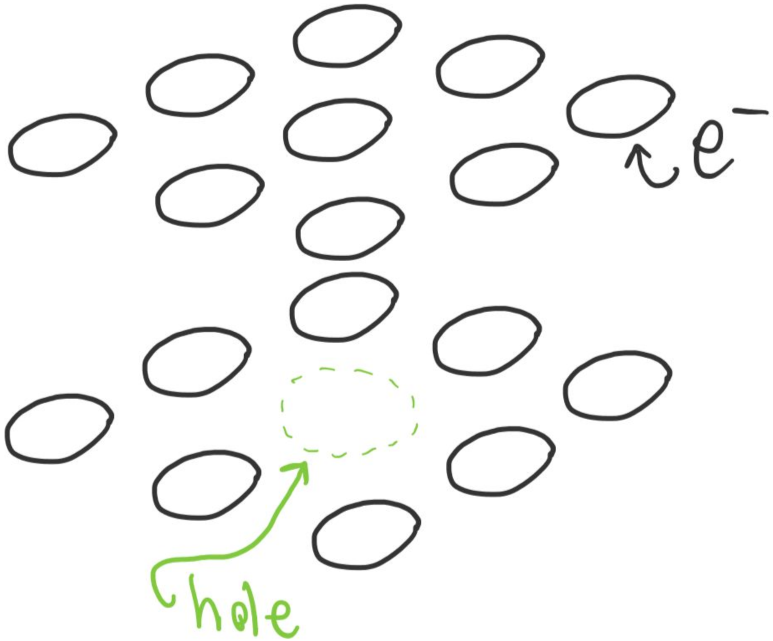
Magnetism

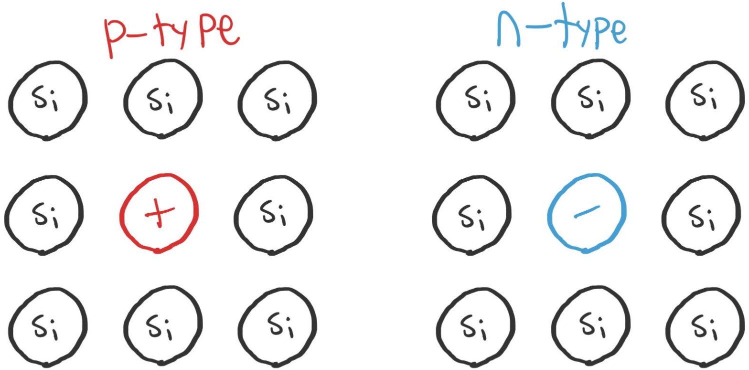
Which of the following is True about Ferromagnets:

I. The spins of neighboring electrons are always aligned.
II. Ferromagnets retain a strong magnetization after saturating in an applied magnetic field.
III. Ferromagnetism is temperature dependent.

A. I only.
B. II only.
C. III only
D. I and II.
E. I, II, and II.

Device Basics (Section 8.2.3)





$$e^- + h^+ = photons + phonons$$

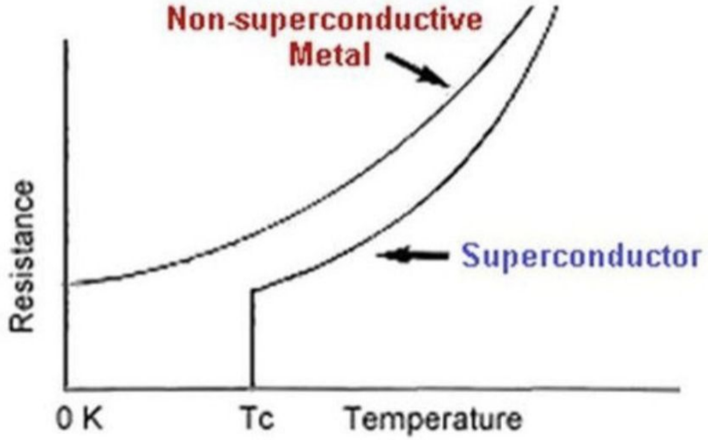
Superconductors

Perfect conductance (zero resistance):

At the critical temperature (Tc) the resistance drops to 0.

Meissner Effect:

At the critical temperature (Tc) the resistance drops to 0. Works through induced surface currents perfectly cancelling the applied magnetic field.



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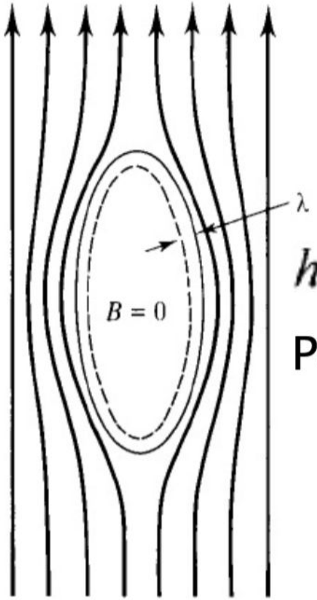
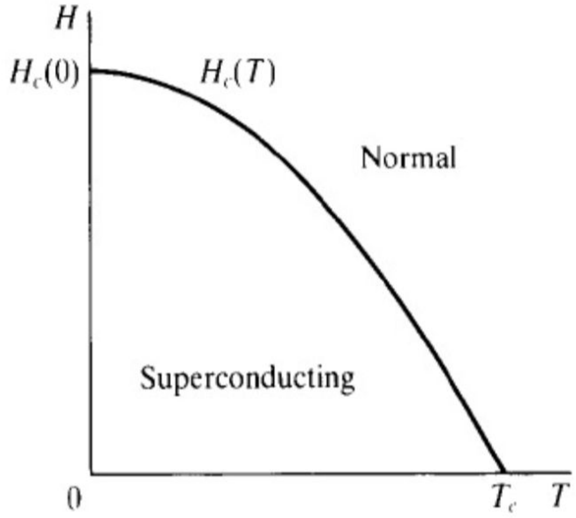


$$h(x) = h(0)e^{-x/\lambda}$$

Penetration depth: λ

Superconductors

$$H_c(T) \approx H_c(0)[1 - (T/T_c)^2]$$



Meissner Effect cont.: A Tale of Two Types

Type I: More common type of SC, the only one referenced in the GRE prep book.

Type II: Less common, more field leakage at low temperatures, but less at higher temperatures. Flux penetrates in quantized amounts.

Superconductors

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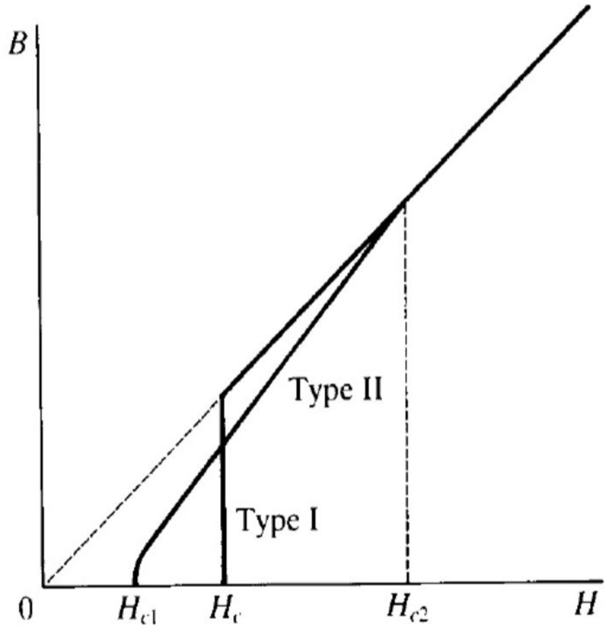

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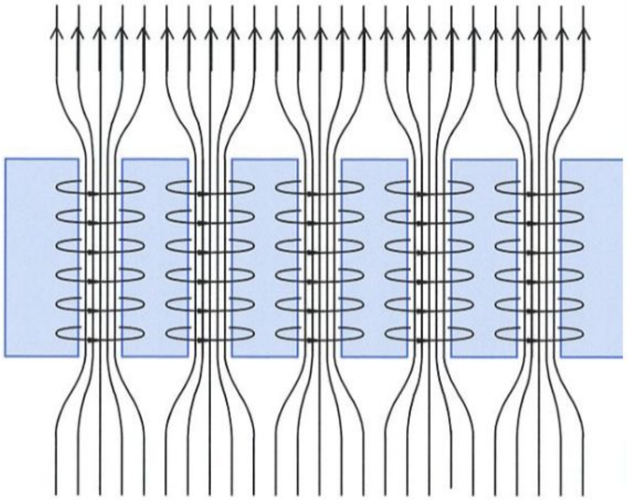


Flux vortices in the mixed state

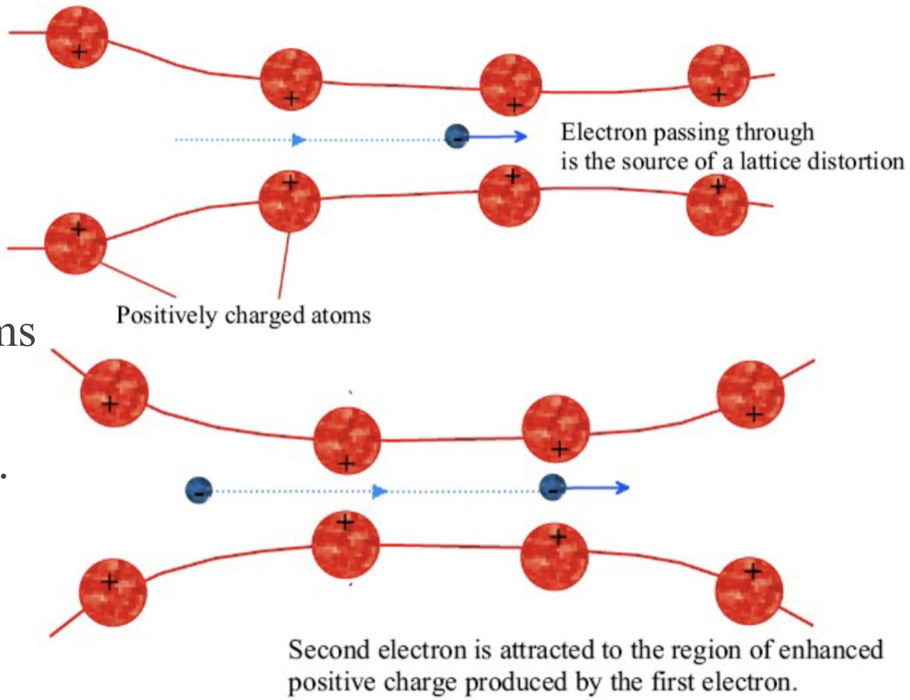
Superconductors

BCS Theory: Cooper Pairs

Defines the principles for the simplest forms of SC. Relies on the electron-phonon interactions that occur at low temperatures.

Low T => quantized phonons ie - small enough lattice vibrations for measurable lattice deformation.

CP energy is below Fermi Energy (electron energy at T = 0K).



Superconductors

Which of the following is True about BCS Superconductors:

I. When $T<T_c$, superconductors expel all applied magnetic field.
II. When $T<T_c$, superconductors have zero resistance.
III. Cooper pairs form due to electron-phonon interactions.

A. I only.
B. III only
C. I and II.
D. II and III.
E. I, II, and II.