Lab Methods

Log Plots

- straight line on log-log plots corresponds to power law: $y = ax^b$
- straight line on log plots (logarithmic y-axis & linear x-axis) corresponds to an exponential growth law: $y = C \cdot A^{Bx}$
- straight line on log-linear plots (linear y-axis & logarithmic x-axis) corresponds to logarithmic growth: $y = C\log(bx)$

Statistics

- Variance: $\underbrace{\sigma_s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2}_{\text{sample variance}}$ OR $\underbrace{\sigma_s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}_{\text{population ("true") variance}}$

- Error:
  - $\sigma_{tot} = \sqrt{\sigma_{stat}^2 + \sigma_{sys}^2}$
  - Error propagation: For $z(x_1, x_2, \ldots, x_n)$, $\sigma_z^2 = \sum_{i=1}^{n}\left(\frac{\partial z}{\partial x_i}\right)^2 \sigma_{x_i}^2$ (probably won't need)
    - Special cases:

$z = ax$ $\qquad\qquad$ $\sigma_z = a\sigma_x$

$z = x \pm y$ $\qquad\qquad$ $\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$

$z = xy$ or $z = \frac{x}{y}$ $\qquad\qquad$ $\frac{\sigma_z}{z} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$

- Weighted averages (e.g. between $x \pm \sigma_x$ & $y \pm \sigma_y$):

$$X = \frac{\frac{x}{\sigma_x^2} + \frac{y}{\sigma_y^2}}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}} \quad \& \quad \sigma_{tot}^2 = \frac{1}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}}$$

2. Event A is drawn from a Gaussian probability distribution with standard deviation σ_A, and event B is drawn from a Gaussian with standard deviation σ_B. If A and B are independent events, the probability distribution for the sum of A and B is a Gaussian with standard deviation

(A) $\sigma_A + \sigma_B$

(B) $\sqrt{\sigma_A \sigma_B}$

(C) $\boxed{\sqrt{\sigma_A^2 + \sigma_B^2}}$

(D) $\dfrac{1}{1/\sigma_A + 1/\sigma_B}$

(E) none of these

Recall: $z = x \pm y$ $\quad \sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$

Distributions

3. Which of the following probability distributions best describes the probability of obtaining heads 3 times when a fair coin is flipped 10 times?

$\boxed{\text{(A) Binomial distribution}}$

(B) Gaussian distribution

(C) Student's *t* distribution

(D) Log-normal distribution

(E) χ^2 distribution

Binomial distribution: probability of obtaining n successes in a fixed number of trials w/ a binary outcome

- Gaussian distribution a good approximation when the success probability is close to 50% & the number of trials is large

Distributions

- Poisson distribution: probability of counting n events in a fixed time, where events occur randomly at a known constant rate
  - $P(n) = \frac{\lambda^n e^{-\lambda}}{n!}$ where λ = expected number of counts in a given time interval
  - $\sigma \approx \sqrt{N}$ for large N
  - $P(0) = e^{-\lambda}$
  - Time between Poisson events follows an exponential distribution
    - Poisson waiting time: $P(t) = \lambda e^{-\lambda t}$

7. A student holding a Geiger counter near a radioactive sample hears five clicks in a 10-second time window. Based on this measurement, what is the probability of hearing exactly one click in a subsequent 10-second time window?

(A) e^{-5}

(B) $5e^{-5}$

(C) $5e^{-2}$

(D) $\dfrac{5e^{-2}}{2}$

(E) 2^4e^{-5}

$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

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Expected value in 10s window: $\lambda = 5$

What is $P(n=1)$?

$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

$$\hookrightarrow P(n=1, \lambda=5) = \frac{5^1 e^{-5}}{1!} = \underline{\underline{5e^{-5}}} \qquad \boxed{B}$$