Main Ch. 20 Equations, an analogy
More on degrees of freedom (d.o.f.), f: Quantum Mechanics
Antibiotic Processor, pt. II

Chapter 20:
* Equipartition Thm: \( \frac{1}{2} kT \) per d.o.f., per molecule
* \( E_{\text{th}} = \frac{f}{2} kT \) per molecule
* \( (KE) = \frac{3}{2} kT \) per molecule \( \Rightarrow \frac{1}{2} MV_{\text{rms}}^2 = \frac{3}{2} kT \)
  \( V_{\text{rms}} = \sqrt{\frac{3kT}{M}} \)

\( f \): d.o.f.

\( E_{\text{th}} = n C_V T \)

\( C_V = \frac{3}{2} R \) for monatomic

Diatomic:
\[ C_V = \begin{cases} \frac{3}{2} R & \text{low} \\ \frac{5}{2} R & \text{med} \\ \frac{7}{3} R & \text{high} \end{cases} \]
Is the following graph of heat capacity vs. temperature that of a monoatomic gas or a diatomic gas?

A) Monoatomic  B) Diatomic  C) Not enough info

Suppose I have 10 J of energy that I want to give to a sample of air via heat. In order to get the largest temperature change, should I give this heat to the gas at 50 K or at 300 K?

A) 50 K  B) 300 K  C) no diff
For a monoatomic gas, what fraction of its total internal energy is translational kinetic?

A) 33.3%  
B) 50.0%  
C) 66.7%  
D) 100%  
E) Depends on the temperature.

An ideal diatomic gas, with molecular rotation but without any vibration, loses a certain amount of energy as heat $Q$. Is the resulting decrease in the internal energy of the gas greater if the loss occurs in a constant-volume process or in a constant-pressure process?

A) Constant-volume process  
B) Constant-pressure process  
C) No difference  

ANS: A
Adiabatic Processes

\[ E_{\text{th}} = nC_vT = \frac{f}{2}nRT = \frac{f}{2}NkT \]

\[ C_v = \frac{f}{2}R \]

\[ C_p = C_v + R = \frac{f+2}{2}R \]

\[ \gamma = \frac{C_p}{C_v} = \frac{f+2}{f} \]

<table>
<thead>
<tr>
<th>( f )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( \frac{5}{3} \approx 1.67 )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{7}{5} = 1.40 )</td>
</tr>
<tr>
<td>7</td>
<td>( \frac{9}{7} \approx 1.29 )</td>
</tr>
</tbody>
</table>
Adiabatic Processes

\[ \Delta U = q + w \]
\[ \Delta U = nC_v \Delta T \]
\[ -pdV = nC_v \Delta T \]

\[ P = \frac{nRT}{V} \]
\[ \Rightarrow \quad -\frac{nRT}{V} \Delta V = nC_v \Delta T \]

\[ T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \]

\[ P_i V_i^\gamma = P_f V_f^\gamma \]

\[ \gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} \]
\[ P V^\gamma = \text{constant} \]

\[ 1 < \gamma \leq \frac{5}{3} \]

<table>
<thead>
<tr>
<th>( f )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( \frac{5}{3} \approx 1.67 )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{7}{3} = 1.4 )</td>
</tr>
<tr>
<td>7</td>
<td>( \frac{9}{7} \approx 1.29 )</td>
</tr>
</tbody>
</table>
Take gas \((\gamma = 1.4)\) and compress quickly to half its initial volume. (adiabatically)

(a) What's \(\left(\frac{P_f}{P_0}\right)\)?

(b) What's \(\left(\frac{T_f}{T_0}\right)\) ?

(c) Would answers true or dec. if we used monatomic gas?

\[
\text{(a) } P_fV_f^\gamma = P_0V_0^\gamma, \text{ with } V_f = \frac{1}{2}V_0 \\
P_f = (2)\gamma P_0 = (2^{1.4})P_0 \approx 2.64P_0
\]

\[
\frac{P_f}{P_0} = 2.64
\]

\[
\frac{T_f}{T_0} = T_f V_f^{\gamma-1} \\
T_0 V_0^{\gamma-1} = T_f V_f^{\gamma-1}
\]

(b) \(T_f = T_0 \left(\frac{V_0}{V_f}\right)^{\gamma-1} = T_0 (2)^{0.4} \approx 1.32T_0\)

\[
\frac{T_f}{T_0} \approx 1.32
\]

(c) \(P\)

\[
V_f = \frac{3}{2}V_0
\]

(This is like \(300K \to 400K\), so you can heat a gas up quite a bit!)