1 Problem 1: True/False

a) True. Air pressure is 1 atm = 1.01 × 10^5 N/m^2 so the force exerted by the weight of the air is \( F = PA = (1.01 \times 10^5 \text{ N/m}^2)(1 \text{ m}^2) = 1.01 \times 10^5 \text{ N} \). The gravitational weight of the car is “only” \( mg = (1000 \text{ kg})(9.81 \text{m/s}^2) = 9.81 \times 10^3 \text{ N} \).

b) False. The ice cube displaces a volume of water equal to its mass, and when it melts it will become water with the same mass and so (since the density of water is a constant) the volume it becomes must just be the same original volume it was displacing—the water level doesn’t change at all.

c) False. The definition of sound intensity level is \( \beta \equiv (10 \text{ dB}) \log_{10} \left( \frac{I}{I_0} \right) \) with \( I_0 \equiv 10^{-12} \text{ W/m}^2 \). Plugging in \( \beta = 41 \text{ dB} \), 99 dB and solving for the intensity of each gives \( I_1 = 10^{-7.9} \text{ W/m}^2 \) and \( I_2 = 10^{-2.1} \text{ W/m}^2 \). While the ratio of these two intensities is \( 10^{5.8} \), the change of absolute intensity is only \( I_2 - I_1 \approx 10^{-2.1} \text{ W/m}^2 \), a much smaller increase than \( 10^5 \text{ W/m}^2 \).

d) True. The intensity follows an inverse square law (\( I \propto \frac{1}{r^2} \)) so if we double the distance from the source the intensity is cut by a factor of \( 1/2^2 = 1/4 \). Since the intensity is proportional to the square of the amplitude (\( I \propto A^2 \rightarrow A \propto \sqrt{I} \)) then decreasing intensity by a factor of 4 means decreasing the amplitude by a factor of 2.
2 Question 2: Cubes and Tension

a) The buoyant force is \( F_B = \rho_f V_{\text{disp}} g \). The cubes are identical and so they have the same volume, meaning they displace the same volume of water. So, the buoyant force on each cube is the same.

b) The cubes are in equilibrium so it must be the case that the upward forces of tension and buoyancy cancel their weight. Written as a force equation for block \( i \): \( F_{\text{net}} = T_i + F_B - m_i g = 0 \rightarrow T = mg - F_B \). In the previous question, we found the buoyant forces to be the same, and furthermore we know that the lead block has more mass as the density of lead is higher than that of aluminum. So, \( mg \) is larger for the lead and \( F_b \) is the same for each meaning the tension holding up the lead block is larger.

c) The cubes being identical means their faces are the same size, so the force on the bottom will be determined by whichever has the larger pressure there. The hydrostatic pressure equation tells us \( p = p_0 + \rho g d \) and so the pressure at the bottom face of the lead cube is larger since it’s deeper. Therefore, the force on the lead cube’s bottom face is larger.

d) We’re given \( T \) and \( \rho_{\text{lead}} \) so we wish to find the volume in terms of those and known constants. The mass of the lead block is \( m = \rho_{\text{lead}} V \). Our force equation told us \( T = mg - F_B \). Plugging in for \( m \) and \( F_B \) we have

\[
T = (\rho_{\text{lead}} V) g - \rho_{\text{water}} V g = (\rho_{\text{lead}} - \rho_{\text{water}}) g V
\]

So solving for \( V \) gives

\[
V = \frac{T}{(\rho_{\text{lead}} - \rho_{\text{water}}) g}
\]

(Recall the boxes have the same volume.)
\section*{3 Question 3: Fluid in a Pipe}

\begin{itemize}
\item[a)] The problem tells us the volume flow rate for the pipe is 1.20 m$^3$/s meaning that, from the continuity equation, everywhere in the pipe we have $A v = 1.20$ m$^3$/s. So, in this section of the pipe where the radius is 0.150 m, the area is $\pi r^2$ and so the velocity is
\[ v = \frac{1.20 \text{ m}^3/\text{s}}{\pi r^2} \approx 17 \text{ m/s}. \]
\item[b)] Using the above analysis for the new radius $r = 0.3$ m we can find the new velocity $v_2 \approx 4.24$ m/s. Applying Bernoulli’s equation to the two points (the height terms cancel because there is “no appreciable change in height”) gives us
\[
p_1 + \frac{1}{2} \rho v_1^2 + \rho g h = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h
\Rightarrow p_2 - p_1 = \frac{1}{2} \rho (v_1^2 - v_2^2)
\]
Plugging in our values for velocity and using $\rho = 1000$ kg/m$^3$ for water, we find that
\[ p_2 - p_1 = 1.35 \times 10^5 \text{ Pa} \]
As a check, we know faster moving fluids exert lower pressure; so, in the larger area with the slower fluid we should have a higher pressure. The answer above tells us the new pressure is around 1.3 atm larger than at the first point, passing the check.
\end{itemize}
4 Question 4: Waves

a) From the snapshot graph, we can see a full oscillation happens for the $x = 0$ particle between $t = 0$ and $t = 0.04$ meaning the period of a full oscillation is $T = 0.04$ s. Likewise, we can see the maximum displacements of each particle are $\pm 4$ mm so the amplitude of oscillation is $A = 4$ mm.

b) We are told the wave travels left, so that the $x = 0.09$ (blue) wave should be leading the $x = 0$ (red) wave. So, when calculating the phase difference between the two, we should do so from blue $\rightarrow$ red. Graphically we find the two differ by a time of about 0.015 s (for example, the blue wave crosses at $t = 0.025$ and the red wave does the same crossing at $t = 0.04$ giving a difference of 0.015). That is, it takes 0.015 s for a displacement in the wave to propagate from $x = 0.09$ to $x = 0$. Since we know the distance between the two points is 0.09 m and the time it takes is 0.015 s we can calculate the velocity of the wave as $v = \Delta x / \Delta t = (0.09 \text{ m}) / (0.015 \text{ s}) = 6 \text{ m/s}$. Since we know $T$, we can find the frequency as well: $f = 1 / T = 25 \text{ Hz}$ and so we can find the wavelength via $v = \lambda f$ by plugging in $v$ and $f$. We find that $\lambda = (6 \text{ m/s}) / (25 \text{ Hz}) = 0.24 \text{ m}$.

c) All that’s left is to solve for $k$, $\omega$, and $\phi_0$. We find $k = \frac{2\pi}{\lambda} = 25\pi / 3 \text{ rad/m}$, and $\omega = 2\pi f = 50\pi \text{ rad/s}$. Using our value of $A$ from part a) we construct

$$y(x,t) = (4 \text{ mm}) \sin \left[ \left( \frac{25\pi \text{ rad}}{3 \text{ m}} \right) x + \left( \frac{50\pi \text{ rad}}{\text{s}} \right) t + \phi_0 \right]$$

(The plus sign is because the wave moves left.) We can solve for $\phi_0$ by plugging in $x = 0, t = 0$; our snapshot graph (for the red line) tells us the value of the displacement at $x = 0, t = 0$ is 0. So

$$0 = 4 \sin (0 + 0 + \phi_0) \rightarrow$$
$$0 = \sin (\phi_0) \Rightarrow \phi_0 = 0$$

Therefore our full answer is

$$y(x,t) = (4 \text{ mm}) \sin \left[ \left( \frac{25\pi \text{ rad}}{3 \text{ m}} \right) x + \left( \frac{50\pi \text{ rad}}{\text{s}} \right) t \right].$$
5 Sound Waves

a) In general, the surface area of the hemisphere is half of the surface area of a full sphere, meaning \( A(r) = 2\pi r^2 \). So for \( r = 25 \text{ m} \) the intensity is

\[
I = \frac{P}{A} = \frac{2 \text{ W}}{1250\pi \text{ m}^2} = 5.1 \times 10^{-4} \text{ W/m}^2.
\]

this corresponds to a dB level of

\[
\beta = (10 \text{ dB}) \log_{10} \left( \frac{5.1 \times 10^{-4} \text{ W/m}^2}{10^{-12} \text{ W/m}^2} \right) = 87 \text{ dB}.
\]

b) The person should run away from the source and our Doppler shift equation for a stationary source, moving observer tells us (if we use the speed of air as \( v = 343 \text{ m/s} \)):

\[
(432 \text{ Hz}) = \left(1 - \frac{v_0}{343 \text{ m/s}}\right)(440 \text{ Hz}) \Rightarrow v_0 = \frac{343 \text{ m/s}}{55} \approx 6.24 \text{ m/s}.
\]