(9 points, 3 points each): 3 Multiple-choice questions / fill-in-the-blanks on various topics.

Directions for multiple-choice questions: COMPLETELY FILL THE SQUARE for the best answer.
Directions for fill-in-the-blank questions: Your answer should be entirely in the boxed region. Include the number of significant figures ("sig. figs.") requested in the problem.

1. A transverse wave is described by the equation \( y(x, t) = (25 \text{ cm}) \cos [(10.0 \text{ m}^{-1})x + (2.0 \text{ sec}^{-1})t] \).
Which of the following is FALSE regarding the properties of this wave on a string?

- The wave is moving in the \((-\hat{x})\)-direction with a speed 0.2 m/s.
- The wave has wavelength 63 m and wavenumber 0.10 per meter.
- The wave has natural frequency 0.32 Hz and amplitude 0.25 meters.
- The wave has angular frequency 2.0 rad/sec and period 3.14 sec.
- The maximum speed of a point on the string is 0.5 m/s and the maximum acceleration of a point on the string is 1.0 m/s².

2. A transverse wave is traveling to the right with velocity 2 m/s and wavelength 4 m (shown below at \( t = 0 \text{ sec} \)). Of the following \((A > 0)\), what will the wave look like at \( t = 1.5 \text{ seconds} \)?

- \(+A \sin(kx)\)
- \(-A \sin(kx)\)
- \(+A \cos(kx)\)
- \(-A \cos(kx)\)
- None of the above.

3. Water flows through a pipe of diameter 2.0 cm such that it fills a 2.0-Liter bottle in 1.00 minute. How fast is water moving through the pipe? (2 sig. fig. answer, in m/s)

\[ V = \frac{\text{vol. water}}{\text{time}} \]

\[ V = \frac{2.0 \times 10^{-3} \text{ m}^3}{60.0 \text{ sec}} = 0.11 \text{ m/s} \]
(a) What is the buoyant force on the ore (when totally immersed in water)?
(b) Find the total volume of the sample.
(c) If the ore is placed in a pool of liquid mercury (density $13.5\rho_w$), the ore would float. What fraction of the total volume of ore would be submerged in the mercury?

(a) \[ T = \frac{3}{5} Mg \text{ (given)} \; ; \; \sum F_z = 0. \]
and so \[ F_{\text{buoy}} = \frac{3}{5} Mg. \]

(b) \[ F_{\text{buoy}} = \frac{3}{5} Mg = \rho_w g V_{\text{disp}} \implies V_{\text{disp}} = \frac{2M}{5\rho_w}. \]

(c) \[ \text{equilibrium} \implies Mg = \rho_{\text{Hg}} g V_{\text{disp}}^{(\text{Hg})}. \]

Therefore, \[ \frac{5}{27} \approx 18.5\% \] of the ore is submerged.
(8 points): The relationship between atmospheric pressure \((P_0)\), gauge pressure \((P_g)\), and absolute pressure \(P\) is

\[ P = P_0 + P_g \]

At one point in a pipeline the water’s speed is 3.00 m/s and the gauge pressure is \(5.00 \times 10^4 \text{ Pa}\). Find the gauge pressure at a second point in the line, 11.0 m lower than the first, if the pipe diameter at the second point is twice that at the first.

\[ \text{(area quadruples } ightarrow \text{velocity down by factor of 4)} \]

\[ V_B = \frac{1}{4} V_A \text{ from continuity} \]

Bernoulli:

\[ p^{(a)} + \frac{1}{2} \rho v_A^2 + \rho g y_A = p^{(b)} + \frac{1}{2} \rho v_B^2 + \rho g y_B \]

\[ p_g^{(a)} + \frac{1}{2} \rho v_A^2 + \rho g H = p_g^{(b)} + \frac{1}{2} \rho \left[ \frac{1}{4} v_A \right]^2 + 0 \]

\[ p_g^{(b)} = p_g^{(a)} + \frac{15}{32} \rho v_A^2 + \rho g H \]

\[ = 5.00 \times 10^4 \text{ Pa} + \frac{15}{32} \left(1000 \text{ m}^3/\text{s}^3 \right) (3.00 \text{ m/s})^2 \]

\[ + \left(1000 \text{ m}^3/\text{s}^3 \right) (9.8 \text{ m/s}^2) (11.0 \text{ m}) \]

\[ p_g^{(b)} = 1.62 \times 10^5 \text{ Pa} = 1.60 \text{ } P_{\text{atm}} \]
6. (15 points, 5 points each): A transverse wave travels in the +x direction with speed 3 m/s. A particle at \(x = 1\) m has displacement vs. time which follows the equation

\[
D_{[x=1\,\text{m}]}(t) = \left(1\,\text{mm}\right) \sin \left[\left(\frac{3\pi}{2}\,\text{rad/seg}\right) t + \pi\right]
\]

You may take all values as exact.

(a) What is the wavelength of this wave?

(b) Draw a snapshot of the wave at \(t = 0\) s (i.e., a plot of \(D(x)\)).

(c) Write down an equation for the wave valid for all \(x\) and \(t\). Have your equation be of the form

\[
D(x, t) = \left(1\,\text{mm}\right) \sin (\ldots)
\]

(a) \(\lambda = \frac{V}{f} = \frac{2\pi V}{\omega} = \frac{2\pi (3\,\text{m/s})}{(3\pi/2\,\text{rad/sec})} = \boxed{4\,\text{m}}\)

(b) 

(c) 

\[
D(x, t) = \left(1\,\text{mm}\right) \sin \left[ \frac{3\pi}{5} x - \frac{3\pi}{5} t + \frac{3\pi}{5} \right] 
\]

We already knew \(k = \frac{2\pi}{\lambda} = \frac{\pi}{2}\,\text{rad/m}\) and \(\omega = \frac{3\pi}{2}\,\text{rad/sec}\).

To find \(\phi_0\), note from the snapshot that at \(t = 0\), the equation should look like \(A\sin(kx - \frac{3\pi}{5}) = A\sin(kx + \frac{3\pi}{5})\).

Thus \(\phi_0 = \frac{3\pi}{2}\) (or \(-\frac{3\pi}{2}\)) and

\[
D(x, t) = \left(1\,\text{mm}\right) \sin \left[ \frac{3\pi}{5} x - \left(\frac{3\pi}{5} \frac{3\pi}{5}\right) t + \frac{3\pi}{5} \right]
\]
(c) Note, because \( \sin \theta = \sin(\pi - \theta) \), you can also write the answer as

\[
D(x,t) = (1\text{mm}) \sin \left[ \pi - (kx - \omega t + \frac{3\pi}{2}) \right]
\]

\[
= (1\text{mm}) \sin \left[ \omega t - kx - \frac{\pi}{2} \right]
\]

\[
= (1\text{mm}) \sin \left[ \omega t - kx + \frac{3\pi}{2} \right]
\]

Concidentally, this is the same phase constant as if you write it as \( A \sin[kx - \omega t + \phi_0] \)!
(10 points, 5 points each): A loudspeaker, broadcasting sound waves at a single frequency, emits sound isotropically.

(a) What is the speaker’s power output if the sound intensity level is 90 dB at a distance of 20 m?
(b) Someone runs straight toward and passes the speaker at constant speed 5.00 m/s. After passing the loudspeaker, the runner hears sound waves at a frequency 8.00 Hz less than when the runner was running towards the speaker. What is the frequency of sound waves emitted by the source?

(a) $90 \text{ dB} = 10^{-3} \frac{W}{m^2} \text{ in SI units}

$I = \frac{P}{A} \Rightarrow P_{\text{source}} = IA = (10^{-3} \frac{W}{m^2})(4\pi)(20m)^2$

$P_{\text{source}} = 5.0 \text{ Watts}$

(b) $f_{\text{before passing}} = 8.00 \text{ Hz} + f_{\text{after passing}}$

$f_0 \left[ 1 + \frac{V_0}{V} \right] = 8.00 \text{ Hz} + f_0 \left[ 1 - \frac{V_0}{V} \right]$

$f_0 \left( \frac{\Delta V_0}{V} \right) = 8.00 \text{ Hz} \Rightarrow f_0 = (8.00 \text{ Hz}) \frac{(345\%)}{2(5.00\%)}$

$f_0 = 274 \text{ Hz}$