1. (15 points, 5 points each): The following graph shows a new kind of car engine. Suppose that \((p, V) = (P_0, V_0)\) at point \(a\), and \((p, V) = (2P_0, 2V_0)\) at point \(c\). A diatomic gas with rotational but not vibrational modes excited (throughout the entire cycle) moves from \(a\) to \(b\) to \(c\) and straight back to \(a\) as shown:

Your answers to the following should be in terms of \(P_0\) and \(V_0\) alone.

(a) Find the heat added to the gas for all three steps of the cycle (positive if heat is added to the gas, negative if heat leaves the gas).

(b) Find the efficiency of this engine.

(c) Find the maximum efficiency possible for any engine running between these two temperatures.

(a) \[ Q_{ab} = nC_v \Delta T = n \left( \frac{5}{2}R \right) \left[ \frac{(2P_0)(V_0) - P_0V_0}{nR} \right] = \frac{5}{2} P_0 V_0 \]

\[ Q_{bc} = nC_p \Delta T = n \left( \frac{7}{2}R \right) \left[ \frac{(2P_0)(2V_0) - P_0V_0}{nR} \right] = \frac{7}{2} P_0 V_0 \]

\[ Q_{ca} = \frac{5}{2} P_0 V_0 - \frac{5}{2} P_0 V_0 - 7 P_0 V_0 = -9 P_0 V_0 \]

(b) \[ \eta = \frac{W_{out}}{Q_H} = \frac{W_{out} + Q_{bc}}{Q_{ab} + Q_{bc}} = \frac{\frac{5}{2} P_0 V_0}{\frac{19}{2} P_0 V_0} = \frac{1}{19} \]

(c) \[ \eta_{max} = 1 - \frac{T_c}{T_H} = 1 - \frac{1}{4} = \frac{3}{4} \]

Note \( T_c = 4T_a \) since \((PV)_c = 4(PV)_a\)
2. (20 points, 5 points each): A refrigerator operates via the following 3-step cycle:

- An isothermal expansion to four times its initial volume. \((A \rightarrow B)\)
- An adiabatic compression that cuts the volume in half. \((B \rightarrow C)\)
- An isobaric process that cuts the volume in half again, returning to the initial point. \((C \rightarrow A)\)

(a) Draw this cycle on a (labelled!) \(pV\) diagram. Call the initial volume and pressure \(V_0\) and \(p_0\).

(b) Solve for the adiabatic constant for this gas. This value is actually impossible (in 3D) – explain why.

(c) Find the Carnot coefficient of performance for any fridge operating between these two temperature extremes.

(d) You don’t have to solve for the actual coefficient of performance of this fridge\(^1\), but explain which heat(s) from which step(s) you would use to plug into \(Q_C\) for the formula \(K = Q_C/W\).

\[\begin{align*}
\text{(a)} & \quad P \quad A \quad C \quad B \\
\text{\(\frac{1}{4}p_0\)} & \quad V_0 \quad 2V_0 \quad \frac{1}{2}V_0 \quad 4V_0 \\
\text{(b) Adiabatic} & \quad \Rightarrow \quad PV^\gamma = \text{const.} \\
& \quad \left(\frac{1}{4}p_0\right)(4V_0)^\gamma = \left(p_0\right)(2V_0)^\gamma \\
& \quad \frac{1}{4}(4)^\gamma = 2^\gamma \\
& \quad 2^\gamma = 4 \Rightarrow \boxed{\gamma = 2} \\
\end{align*}\]

\(\gamma = 2\) is impossible... the largest \(\gamma\) can be is \(\frac{5}{3}\) (monatomic gas w/ only 3 d.o.f.)

\[\begin{align*}
\text{(c) } K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} = \frac{\frac{5}{3}T_H}{T_H - \frac{5}{3}T_H} = \boxed{1} \\
\text{(note } T_H = T_\text{Air} \text{ is twice the value at } T_C = T_\text{Air} & \text{A and B)} \\
\text{since } pV_C = 2p_A V_A) \\
\end{align*}\]

\(\text{(d) Heat is delivered from the cold reservoir to the gas during the } A \rightarrow B \text{ step. } \boxed{Q_C = Q_A - B}. \text{ This is the only step for which } Q > 0.\)

\[\begin{align*}
\end{align*}\]

\(^1\)Actually, you’d get a value bigger than \(K_{\text{Carnot}}\) because of the issue raised in part (b).
3. (10 points, 5 points each): The magnetic field of a linearly-polarized EM wave is described by the following equation:

\[ B(y, t) = (1 \mu T)(+\hat{z})\cos(ky + \omega t) \]

The wavelength of the wave is 540 nm (green light).

(a) What is the frequency of this wave, in Hertz?

(b) Find the amplitude of the electric field. Also, find the direction of the electric field at \( x = 0 \) and \( t = 0 \).

\[ (a) \quad f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{m/s}}{540 \times 10^{-9} \text{m}} = 5.6 \times 10^{14} \text{Hz} \]

\[ (b) \quad E_0 = cB_0 = (3.00 \times 10^8 \text{m/s})(1 \times 10^{-6} \text{T}) = 300 \text{ V/m} \]

\[ \vec{E} \times \vec{B} \] should point in the \(-\hat{y}\) direction

\[ (\vec{?}) \times (+\hat{z}) = (-\hat{y}) \]

\[ (\vec{?}) = (+\hat{x}) \quad (\text{dir. of } \vec{E} @ y=0 \text{ and } t=0) \]
4. (15 points, 5 points each): A double-slit experiment yields the pattern of closely spaced bright and dark fringes shown below. Only the central portion of the pattern is shown in the figure. The bright spots are equally spaced at 1.53 mm center to center (except for the missing spots) on a screen 2.50 m from the slits. The light source was a He-Ne laser producing a wavelength of 632.8 nm.

(a) How far apart are the two slits?
(b) What is the path-length difference between two consecutive bright spots?
(c) Explain three different changes we could make to the setup so that the bright spots appear closer together.

(a) \[ \frac{\Delta y}{L} = \lambda \] (small angle approx is okay since all bright spots are equally spaced)

\[ d = \frac{2L}{\Delta y} = \frac{(632.8 \times 10^{-9} \text{ m})(2.50 \text{ m})}{1.53 \times 10^{-3} \text{ m}} = \boxed{1.03 \text{ mm}} \]

(b) Two consecutive bright spots have \[ |(\Delta r)_1 - (\Delta r)_2| = 2 \]

[The question doesn't really make sense as written... one bright spot has a path-length difference of \( m\lambda \) from the two light rays from the two slits.]

(c) * Increase f (decrease \( \lambda \)) of laser.
* Move screen closer to slits.
* Submerge apparatus in material w/ \( n > 1 \) (e.g., water).
(15 points, 3 points each): 5 Multiple-choice questions / fill-in-the-blanks on various topics.

Directions for multiple-choice questions: COMPLETELY FILL THE SQUARE for the best answer.
Directions for fill-in-the-blanks questions: Your answer should be entirely in the boxed region. Include the number of significant figures ("sig. figs.") requested in the problem.

5. Which of the following statements regarding engines and refrigerators is FALSE?

- It is impossible to construct a fridge that takes energy in the form of heat from a low temperature reservoir and delivers the same amount of energy to a high temperature reservoir.  \( \text{False (2nd Law)} \)
- You can cool the kitchen by leaving the refrigerator door open.  \( \text{False} \) \( \Delta Q_H > 
\Delta Q_C \) and overall work goes to heating up the room.
- You can heat the kitchen by leaving the oven open.
- In a fridge, you can take out more energy in the form of heat than work that you put in.  \( \text{True} \) \( (k > 1) \)

6. The engine in your car takes in heat to do work. For every 100.0 kJ of energy in the form of heat that your car takes in, it does 20.0 kJ of work, and spits out 80.0 kJ in the form of heat. The lower heat reservoir of the engine is 300.0 K. What is the temperature of the hot reservoir, assuming your car is a perfect Carnot engine? If your car is not a perfect Carnot engine, is the hot reservoir actually warmer or colder than this?

- 1500 K, warmer.
- 1500 K, colder.
- 600 K, colder.
- 380 K, colder.
- 380 K, warmer.

\[
\eta_{\text{Carnot}} = \frac{\text{Work}}{\Delta Q_H} = \frac{\Delta Q_C - \Delta Q_h}{\Delta Q_H} = \frac{T_H - T_C}{T_H} = \frac{80}{100.0} = 1 - \frac{300K}{T_H}
\]

\[
T_H = \frac{5}{4}(300K) = 375K
\]

7. Two speakers are emitting sound in phase with one another, and you are standing exactly halfway between the speakers. You hear sound of wavelength 1.00 meter. How far would someone need to move one of the speakers directly away from you in order for you to hear destructive interference? Give the smallest possible distance the speaker has to be moved. (2 sig. fig. answer)

\[
\Delta \gamma = \frac{1}{2} \lambda = 0.50 \text{m}
\]

Move speaker 

[Diagram of speakers and wavelength]
8. Unpolarized light with electric field amplitude $E_0$ passes first through a polarizer with its polarization axis vertical, then through one with its axis 35° to the vertical. What is the electric field amplitude after the second polarizer?

- $\boxed{0.34 E_0}$
- $\boxed{0.58 E_0}$
- $0.67 E_0$
- $0.75 E_0$
- $0.82 E_0$

\[ \frac{I_f}{I_0} = \frac{1}{2} \cos^2 35° = 0.336 \]

\[ E_f = \sqrt{0.336} E_0 \]

9. Which of the following is a possible equation for the magnetic field of a light wave traveling in the $(-\hat{y})$-direction, with a wavelength in the visible part of the EM spectrum, which has its electric field parallel to the $\hat{x}$-axis?

- $\boxed{\vec{B}(x, y, z, t) = (1.00 \text{nT}) \cos \left[ \left( 1.26 \times 10^7 \text{m}^{-1} \right) y + \left( 3.77 \times 10^{15} \text{s}^{-1} \right) t \right] (+\hat{z})}$
- $\vec{B}(x, y, z, t) = (1.00 \text{nT}) \cos \left[ \left( 3.77 \times 10^7 \text{m}^{-1} \right) z - \left( 1.26 \times 10^{15} \text{s}^{-1} \right) t \right] (-\hat{y})$
- $\vec{B}(x, y, z, t) = (1.00 \text{nT}) \cos \left[ \left( 3.77 \times 10^7 \text{m}^{-1} \right) y - \left( 1.26 \times 10^{15} \text{s}^{-1} \right) t \right] (-\hat{z})$
- $\vec{B}(x, y, z, t) = (1.00 \text{nT}) \cos \left[ \left( 1.26 \times 10^7 \text{m}^{-1} \right) z - \left( 3.77 \times 10^{15} \text{s}^{-1} \right) t \right] (+\hat{y})$
- $\vec{B}(x, y, z, t) = (1.00 \text{nT}) \cos \left[ \left( 1.26 \times 10^7 \text{m}^{-1} \right) z + \left( 3.77 \times 10^{15} \text{s}^{-1} \right) t \right] (+\hat{y})$

10. What is your seat number (on the chair you’re sitting in)? This problem is not worth any points, but you will be penalized if you do not fill it out.

In most rooms, Letter Number

\[ J7 \]