1. Given this picture, what is the density of Mercury?

\[ 1 \text{ atm} = 101300 \text{ Pa} = 760 \text{ mm Hg} \]

SI unit: \( \text{Pa} \)

\[ P(d) = P_{\text{top}} + \rho g d \]

pressure vs. depth.

Pressure at (1) = Pressure at (2) = \( \neq 1 \text{ atm} \)

Pressure at (1) = \( P_{\text{top}} + \rho_{\text{Hg}} g d = 1 \text{ atm} = 101300 \text{ Pa} \)

\[ \rho_{\text{Hg}} \times 9.8 (m^2/s^2) \times (0.76 m) = 101300 \text{ Pa} \]

\[ \Rightarrow \rho_{\text{Hg}} = 13600 \text{ kg/m}^3 \]

2. A wood block floats

* in fresh water with 2/3 of its volume is submerged.

*in oil with 90% of its volume is submerged.

(a) Find the density of the wood block.
(b) Find the density of oil.

\[ \text{(a)} \]

\[ \begin{align*}
F_B &= \rho_{\text{water}} g \left( \frac{2}{3} V \right) \\
mg &= F_B = \rho_{\text{water}} g \left( \frac{2}{3} V \right) \\
\rho_{\text{water}} V &= \rho_{\text{water}} g \left( \frac{2}{3} V \right) \\
\rho_{\text{wood}} &= \frac{2}{3} \rho_{\text{water}} \\
\rho_{\text{water}} &\approx 1000 \text{ kg/m}^3
\end{align*} \]

\[ \text{(b)} \]

\[ \begin{align*}
F_B &= \rho_{\text{oil}} g \left( 0.9 V \right) \\
mg &= F_B = \rho_{\text{oil}} g \left( 0.9 V \right) \\
\rho_{\text{wood}} &= 0.9 \rho_{\text{oil}} \\
\frac{2}{3} \rho_{\text{water}} &= 0.9 \rho_{\text{oil}} \\
\rho_{\text{oil}} &= 0.74 \rho_{\text{water}}
\end{align*} \]
3. To suck lemonade of density $983 \text{ kg/m}^3$ up a straw to a maximum height of 4.43 cm, what minimum gauge pressure (in atmospheres) must you produce in your lungs?

$$\text{Equation: pressure vs. depth.}$$

$$p(d) = p_0 + \rho gd$$

$$\text{absolute pressure} \quad \text{atm. pressure} \quad \text{gauge pressure}.$$

The gauge pressure: $\rho gd$.

$$\rho gd = 9.83 \left( \text{kg/m}^3 \right) \times (9.8 \text{ m/s}^2) \times (4.43 \times 10^{-2} \text{ m})$$

$$= 426.8 \text{ Pa}$$

$$\text{Absolute lung pressure} \quad p_{\text{lung}} = 101300 \text{ Pa} - 427 \text{ Pa} = 100873 \text{ Pa}$$

4. The L-shaped tank shown in the figure is filled with water and is open at the top. If $d = 3.23$ m, what is the total force exerted by water (a) on face $A$ and (b) on face $B$?

(a) $F_A = p_A \cdot A_A = \rho_{\text{water}} \cdot g \cdot (2d) \cdot d^2$

$$= 2 \rho_{\text{water}} \cdot g d^2 = 2.04 \times 10^6 \text{ N}$$

By Air pressure is canceled out on 2 sides of the tank.

(b) We can use average pressure at surface $B$.

The average pressure at $B$ is the pressure at the level of the center of surface $B$. We know this due to the fact that pressure vs. depth is a linear function.

$$F_B = p_{\text{ave}} \cdot A_B = \rho_{\text{water}} \cdot g \left( \frac{5d}{2} \right) \cdot d^2 = \frac{5}{2} \rho_{\text{water}} \cdot g \cdot d^3$$

$$= 2.56 \times 10^6 \text{ N}$$
5. What would be the height of the atmosphere if the air density (a) were uniform and (b) decreased linearly to zero with height? Assume that at sea level the air pressure is 1.00 atm and the air density is 1.22 kg/m$^3$.

Equation of pressure vs depth modified: \[ P_z = P_i - \int_0^z \rho g \, dy \]

* For uniform density, \( \rho = \text{const.} \) \( \Rightarrow P_z = P_i - \rho g (h-0) \)

* For air density decreased linearly to zero w/ height, \( \rho = \rho_0 (1 - \frac{y}{h}) \)

where \( \rho_0 \) is density at sea level.

\[ \Rightarrow P_z = P_i - \int_0^h \rho_0 g (1 - \frac{y}{h}) \, dy = P_i - \frac{1}{2} \rho_0 g h. \]

(a) Uniform \( \rho \), \( \Rightarrow P_z = P_i - \frac{1}{2} \rho_0 g h \)

(b) Decrease density linearly, \( \rho_1 = \frac{1}{2} \rho_0 g h \)

\[ \Rightarrow h = \frac{2 P_i}{\rho_0 g} = \frac{2 \times 101300}{1.22 \times 9.8} = 16945 \, \text{m} \]

6. What fraction of the volume of an iceberg (density 917 kg/m$^3$) would be visible if the iceberg floats in (a) the ocean (salt water, density 1024 kg/m$^3$) and (b) in a river (fresh water, density 998 kg/m$^3$)? (When salt water freezes to form ice, the salt is excluded. So, an iceberg could provide fresh water to a community.)

Let \( V_i \) be the total volume of the iceberg and \( V_f \) is the submerged portion.

Fraction that of iceberg that is visible is \( \text{Frac} = \frac{V_i - V_f}{V_i} = 1 - \frac{V_f}{V_i} \)

Iceberg is floating \( \Rightarrow \rho_i g V_i = \rho_f g V_f \)

\[ \Rightarrow \rho_i V_i = \rho_f V_f \]

\[ \Rightarrow \text{Frac} = 1 - \frac{\rho_i}{\rho_f} \]

(a) \( \text{Frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917}{1024} = 0.104 = 10.4\% \)

(b) \( \text{Frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917}{998} = 0.0803 = 8.3\% \)
7. Water is moving with a speed of 5.5 m/s through a pipe with a cross-sectional area of 4.6 cm². The water gradually descends 12 m as the pipe increases to 9.0 cm². (a) What is the speed at the lower level? (b) If the pressure at the upper level is $1.5 \times 10^5$ Pa, what is the pressure at the lower level?

a) Continuity eqn. \[ A_1 v_1 = A_2 v_2 \quad \Rightarrow \quad v_2 = \frac{A_1}{A_2} v_1 \]
\[ v_2 = \frac{4.6 \text{ cm}^2}{9 \text{ cm}^2} \times 5.5 \text{ m/s} \approx 2.81 \text{ m/s} \]

b) Bernoulli eqn. \[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \]
\[ P_2 = P_1 + \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g (h_1 - h_2) \]
\[ = 1.5 \times 10^5 \text{ Pa} + \frac{1}{2} \times 1000 \frac{\text{kg}}{\text{m}^3} \left[ (2.81 \text{ m/s})^2 - (5.5 \text{ m/s})^2 \right] + (1000 \frac{\text{kg}}{\text{m}^3}) \times 0.8 \frac{\text{m}}{\text{s}^2} \times 12 \text{ m} \]
\[ = 2.79 \times 10^5 \text{ Pa} \]

8. A cylindrical tank with a large diameter is filled with water to a depth $D = 0.280$ m. A hole of cross-sectional area $A = 6.92 \text{ cm}^2$ in the bottom of the tank allows water to drain out. (a) What is the rate at which water flows out, in cubic meters per second? (b) At what distance below the bottom of the tank is the cross-sectional area of the stream equal to one-half the area of the hole?

Since the tank is large, we neglect water speed at the top (also water level is quite constant).

a) \[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \]
\[ P_1 = P_2 = P_{\text{atm}}, \quad v_1 = 0 \text{ by assumption above.} \]
\[ \Rightarrow \quad \rho g h_1 = \frac{1}{2} \rho v_2^2 + \rho g h_2 \]
\[ v_2 = \sqrt{2g (h_1 - h_2)} \]

Flow rate \[ A v_2 = A \sqrt{2g (h_1 - h_2)} = (6.92 \times 10^{-4} \text{ m}^2) \sqrt{2 \times (9.8 \text{ m/s}^2)(0.28 \text{ m})} \]
\[ = 1.62 \times 10^{-3} \text{ m}^3/\text{s} \]
b) Continuity eqn. \( A_2 v_2 = A_3 v_3 \)

\[ V_3 = \frac{A_2}{A_3} v_2 = 2v_2 \]

Bernoulli eqn. \( \frac{1}{2} \rho v_2^2 + \rho g h_2 = \frac{1}{2} \rho v_3^2 + \rho g h_3 \)

Same pressure at 2 and 3

\[ h_2 - h_3 = \frac{V_3^2 - V_2^2}{2g} = \frac{4V_2^2 - V_2^2}{2g} = \frac{3V_2^2}{2g} = \frac{3 \times [2g (h_4 - h_e)]}{2g} \]

\[ h_{23} + h_2 - h_3 = 3(h_4 - h_e) = 3 \times 0.28 \text{ m} = 0.84 \text{ m} \]