A transverse sinusoidal wave is moving along a string in the positive direction of an x axis with a speed of 82 m/s. At t = 0, the string particle at x = 0 has a transverse displacement of 4.3 cm from its equilibrium position and is not moving. The maximum transverse speed of the string particle at x = 0 is 20 m/s. (a) What is the frequency of the wave? (b) What is the wavelength of the wave? If the wave equation is of the form \( y(x, t) = A \sin(kx \pm \omega t + \phi) \), what are (c) A, (d) k, (e) \( \omega \), (f) \( \phi \), and (g) the correct choice of sign in front of \( \omega \)?

a) particle max speed \( u_{\text{max}} = \omega A \) \( \Rightarrow \) \( \omega = \frac{u_{\text{max}}}{A} = \frac{20 \text{ m/s}}{0.043 \text{ m}} = 465 \text{ rad/s} \)

\( \omega = 2\pi f \) \( \Rightarrow \) \( f = \frac{\omega}{2\pi} = 74 \text{ Hz} \)

b) \( v = f \lambda \) \( \Rightarrow \) \( \lambda = \frac{v}{f} = \frac{82}{74} = 1.11 \text{ m/s} \)

(c), (d), (e), (f), (g)

\( A = 0.043 \text{ m}, \) \( k = \frac{2\pi}{\lambda} = 5.67 \text{ rad m}^{-1} \)

\( \omega = 465 \text{ rad/s} \)

At \( x = 0, \ t = 0 \) \( \Rightarrow \) \( y(0,0) = A \sin(\phi) = A \) \( \Rightarrow \) \( \sin \phi = 1 \) \( \Rightarrow \) \( \phi = \frac{\pi}{2} + m2\pi \)

\( \Rightarrow \) \( y(x,t) = 0.043 \sin(5.67x - 465t + \frac{\pi}{2}) \)

minus sign due to wave travels in +x direction

A stretched string has a mass per unit length of 5.17 g/cm and a tension of 26.8 N. A sinusoidal wave on this string has an amplitude of 0.190 mm and a frequency of 173 Hz and is traveling in the negative direction of an x axis. If the wave equation is of the form \( y(x,t) = A \sin(kx + \omega t) \), what are (a) A, (b) k, and (c) \( \omega \), and (d) the correct choice of sign in front of \( \omega \)?

a) \( A = 0.19 \text{ mm} \)

b) \( \lambda = \frac{v}{f} = \frac{\sqrt{\frac{2}{\mu}}}{f} \) where \( \lambda \) is tension \( \mu \) is mass density.

\( \Rightarrow \) \( \lambda = \frac{2\pi}{\lambda} = 2\pi f \sqrt{\frac{\mu}{\varepsilon}} = 2\pi (173) \sqrt{\frac{0.517 \text{ kg/m}}{26.8 \text{ N}} = 151 \text{ m}^{-1}} \)

(c) \( \omega = 2\pi f = 1087 \text{ rad/s} \)

\( y(x,t) = (0.19 \text{ mm}) \sin(151x + 1087t) \)

plus sign due to travelling in -x direction.
A transverse wave traveling in the +x direction with speed of 5.0m/s has a crests at both \( x = 3.0\) m and \( x = 5.0\) m at time \( t = 0\). The particles are a distance 0.5mm from when they are at their troughs.

(a) Give three possible wavelengths for this wave.

(b) Given the wavelength of this wave is longest it possibly could as in part (a). Write the equation of the wave \( y(x,t) = A \cos \left[ ... \right] \)

\[
\begin{align*}
\text{a)} & \quad \text{Between 2 crests can be an integer number of wavelengths} \\
& \quad \frac{5(m) - 3(m)}{2(m)} = m \lambda \\
& \quad \Rightarrow \lambda = \frac{2}{m} \quad \Rightarrow \lambda \text{ could be } 2(m), 1(m), 0.5(m), ... \\
\text{b)} & \quad \text{Longest possible } \lambda \text{ is } 2(m).
\end{align*}
\]

\[
\begin{align*}
\omega &= \frac{2 \pi}{\lambda} = \pi \text{ m}^{-1}, \quad \alpha = \frac{v}{\lambda} \Rightarrow f = \frac{v}{\lambda} = \frac{5}{2} = 2.5 \text{ Hz} \\
\Rightarrow \omega &= 2\pi f = 5\pi \text{ rad/s} \quad \Rightarrow \Delta = \frac{0.5 \text{ mm}}{2} = 2.5 \text{ mm} \\
\text{At } & \quad x = 3 \text{ m}, \quad t = 0 \text{s}, \quad y(3,0) = A \\
\Rightarrow \cos \left[ \pi \left( \frac{3}{\lambda} \right) \omega t + \phi \right] &= 1 \quad \Rightarrow \phi = \pm \pi, \pm 3\pi. \text{ etc}
\end{align*}
\]

The linear density of a string is \( 1.8 \times 10^{-4} \text{ kg/m} \). A transverse wave on the string is described by the equation

\[
y = (0.013 \text{ m}) \sin((2.3 \text{ m}^{-1})x + (25 \text{ s}^{-1})t)
\]

What are (a) the wave speed and (b) the tension in the string?

(a) wave speed

\[
v = \frac{\lambda}{T} = \frac{\omega}{k}
\]

From equation \( \lambda = 2.3 \text{ m}^{-1}, \omega = 25 \text{ rad/s} \)

\[
\Rightarrow v = \frac{25}{2.3} = 10.87 \text{ m/s}
\]

(b) \( v = \sqrt{\frac{E}{\mu}} \Rightarrow \zeta = \sqrt{\frac{\mu \nu^2}{E}} \)

\[
\zeta = 1.8 \times 10^{-4} \left( \frac{\text{kg/m}}{m} \right) \times \left[\left(0.87 \text{ m/s}\right)\right]^2 = 0.021 \text{ N}
\]
The upper end of a wire of length $L$ is fastened to the ceiling, and a mass $M$ is suspended from the lower end of the wire. You observe that it takes a transverse pulse time $T$ to travel from the bottom to the top of the wire.

(a) What is the mass of the wire?
(b) Check that your answer in (a) has the correct dimensions.

$$V = \frac{L}{T} \quad \text{wave speed}$$

Also, $V = \sqrt{\frac{Z}{\mu}}$, where tension $Z = Mg$

$$V = \sqrt{\frac{Mg}{\frac{M_{\text{string}}}{L}}} = \sqrt{\frac{MgL}{M_{\text{string}}}} \quad \Rightarrow \quad \frac{L}{T} = \sqrt{\frac{MgL}{M_{\text{string}}}}$$

$$\Rightarrow M_{\text{string}} = \frac{MgL}{L} = Mg$$

(b) $[kg] \div [kg] \left[ \frac{m}{s^2} \right] \left[ s^2 \right] / [m] \rightarrow \text{check!}$

During takeoff, the sound intensity level of a jet engine is $140\text{dB}$ at a distance of $30\text{m}$.

(a) What is the absolute sound intensity at $30\text{m}$?
(b) Your eardrum is $6\text{mm}$ in diameter. How much energy will be transferred to your eardrum for $1$ min.?
(c) What is sound intensity in dB at $300\text{m}$?

(a) $140 = 10 \log \frac{I}{I_o}$

$$\Rightarrow \log \frac{I}{I_o} = 14 \Rightarrow I = 10^{14} I_o = 100 \text{ W/m}^2$$

(b) Eardrum Area $A = \pi d^2$

Power transfer $P = IA = \frac{\pi I d^2}{4}$

Energy in $1$ min $E = Pt = 60(s) \frac{\pi I d^2}{4} = 15\pi I d^2$

$$= 15\pi \times 100 \left( \frac{W}{m^2} \right) \times \left( 6 \times 10^{-3} m \right)^2$$

$$= 0.17 \text{ J}$$

(c) dB at $300\text{ m}$, Intensity at $300\text{ m}$ is $100$ times smaller than at $30\text{ m}$. Due to distance is $10$ times larger.

$$10 \log \left( \frac{I}{I_o} \cdot \frac{1}{100} \right) = 10 \log \left( \frac{I}{I_o} \right) \cdot 10 \log (100) = 10 \log \frac{I}{I_o} - 20 \text{ dB}$$

$$\Rightarrow 120 \text{ dB at } 300\text{ m}$$
Two submarines A and B move toward each other. Sub A moves at speed \(u_A\), and sub B moves at speed \(u_B\). Sub A sends out a sonar signal at frequency \(f\). Sonar waves travel at speed \(c\).

(a) What is the signal’s frequency as detected by sub B?

(b) What is the frequency detected by sub A in the signal reflected back to it by sub B?

Doppler formula: \[ f = f_0 \frac{V + u_D}{V - u_A} \]

\[ f_B = f = f_0 \left( \frac{V + u_B}{V - u_A} \right) \]

Reflected frequency detected by sub A:

\[ f'_2 = f = f_0 \left( \frac{V + u_A}{V - u_B} \right) = f \left( \frac{V + u_A}{V - u_B} \right) \left( \frac{V + u_B}{V - u_A} \right) \]