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Physics 500 (Intro to Teaching Physics), Fall 2018

HOMEWORK 7 (LaTeX)

Write up a problem and solution to a homework problem in LaTeX. The problem is up to you, and could be directed at any level student. (it could even be a homework problem from a class you're taking yourself right now!) Your solution should have an equation in an equation environment, where you have more than one line aligned at an equals sign. Pictures / figures are optional but might clarify your problem / solution (depends on your level of LaTeX – don't get too fancy if you're just starting out). There is an example problem / solution below.

Send your .tex and .pdf files to me via email to submit the assignment. You are free (and encouraged!) to submit HW6 and HW7 together in one email.

Problem: A neutral rod of length L lies on the x-axis from $x = -L/2$ to $x = +L/2$. Despite having no net charge, the rod has charge per unit length $\lambda(x) = 2\lambda_0 x/L$. Find the approximate electric potential for points $x \gg L$ on the $+x$ axis.

Solution: It's clear that the charge density is an odd function and therefore the dipole moment is nonzero. Since the monopole moment is zero and the dipole moment is nonzero, the potential at large distances is dominated by the dipole term in the multipole expansion.

To find the dipole moment $\mathbf{p} = p_x\hat{\mathbf{x}}$, we'll sum up all the tiny dipoles comprised of dq (that exist at points $x > 0$ on the right half of the rod) paired with the opposite charge at $-x$. Note the distance between these two charges is $2x$.

$$\begin{aligned} p_x = \int_{\text{right half of rod}} (2x)(dq) &= \int_{x=0}^{L/2} (2x)\lambda(x)dx \\ &= \int_{x=0}^{L/2} (2x)\left(\frac{2\lambda_0 x}{L}\right)dx \\ &= \frac{4\lambda_0}{L}\int_{x=0}^{L/2} x^2 dx \\ &= \frac{4\lambda_0}{L}\left[\frac{(L/2)^3}{3}\right] = \frac{\lambda_0 L^2}{6} \\ \mathbf{p} &= \frac{\lambda_0 L^2}{6}\hat{\mathbf{x}} \end{aligned}$$

The potential is therefore

$$V(x, \theta = 0) = \frac{kp}{x^2}\cos 0^\circ = \boxed{\frac{k\lambda_0 L^2}{6x^2}}$$